Order of Operations - PEMDAS

Operations are the actions in math. Most operations come in pairs of opposites: addition and subtraction; multiplication and division; and exponents and roots. There is a priority of operations to tell us which action to do first. PEMDAS is a way to remember the order.

PEMDAS stands for Parentheses, Exponents, Multiplication & Division, Addition & Subtraction. So, solve the values inside parentheses first. If there are multiple sets of parentheses, work from the inner parentheses outwards. Second, calculate exponents. Third, do multiplication and division. Finally, do addition and subtraction. Within each priority level, work from left to right. As you practice, write your work directly below the line above it.

For example: 1 + [3 x (2+1)]

1 + [3 x 3]

1 + 9

10

For example: 3 x 2^2 + [(8 + 7) – (5 x 3)]

3 x 2^2 + [15 – 15]

3 x 4 + 0

12

Associative Property for Addition or Multiplication

The associative property means that the terms, the "pieces", of an expression can be grouped together with parentheses ("associated") in any way without changing the value. This only works for addition or for multiplication. It does not work for a combination of addition and multiplication together, and it does not work for subtraction and division at all.

For addition: (8 + 2) + 3 + (9 + 7) =

10 + 3 + 16 =

29

is the same as 8 + (2 + 3 + 9) + 7 =

8 + 14 + 7 =

29

For multiplication: (2 x 3) x (4 x 5) =

6 x 20 =

120

is the same as 2 x (3 x 4) x 5 =

2 x 12 x 5 =

120

Commutative Property for Addition or Multiplication

The commutative property is just for addition or multiplication. The terms can "commute," or move around, in the expression without changing the value.

For addition: 3 + 2 + 7 is the same as 2 + 7 + 3

For multiplication: 1 x 2 x 3 x 4 is the same as 3 x 1 x 4 x 2

Distributive Property for Multiplication and Division

If addition or subtraction is in parentheses, which in turn is being multiplied by a number, we can multiply each term of the addition or subtraction first and THEN add or subtract. And if some addition or subtraction is in parentheses, which is being divided by a number, we can divide each term first and then add or subtract. Look at these examples:

The multiplication form, with letters, looks like: a(b+c) = ab + ac

> You should note: ab means "a times b". When two or more terms are next to each other, it is multiplication.

This example with numbers: 5(2 + 3)

5 x 2 + 5 x 3

10 + 15 = 25

We could have added (2 + 3) first, then multiplied by 5 and gotten the same answer, 25. So why do we bother with this? When we solve equations, some of the terms may contain variables which we may not be able to add first. Distribution is important to solve equations.

The division form, with letters, looks like: $\frac{b+c}{a} = \frac{b}{a} + \frac{c}{a}$. Remember, what looks like a fraction means division.

This example with numbers: $\frac{6+3}{3} = \frac{9}{3} = 3$

Or, with distribution: $\frac{6+3}{3} = \frac{6}{3} + \frac{3}{3}$

2 + 1 = 3

Identities

There are "identity" numbers for each operation that will not change the value of the number. For addition, zero is the identity, because it can be added to any number without changing its value.   3 + 0 = 3.

Zero is also the identity for subtraction: 3 – 0 = 3.

One is the identity for both multiplication and division.   5 x 1 = 5.   And 5 ÷ 1 = 5

These identities may seem obvious, and many people rush past this information without realizing how important it is. The multiplication identity is crucial to the process of converting a fraction to another format to have a common denominator. The sections on fractions and solving equations will be full of examples that use these properties.

Factoring

To "factor" means to break down into smaller components. In math, that means to find the smaller numbers that can be multiplied together to get a larger number. Remember, multiplication and division are opposites of each other.

So, to "factor" 10 is the same as asking "What numbers divide into 10 evenly (without 'remainders')?" We can use some trial-and-error, starting with 1, then 2, then 3 and so on to find all the factors of a number. 10 is evenly divisible by 1, 2, 5 and 10. Some examples:

10 = 1 x 10 = 2 x 5 So the factors of 10 are: 1, 2, 5, 10	12 = 1 x 12 = 2 x 6 = 3 x 4 So the factors of 12 are: 1, 2, 3, 4, 6, 12	7 = 1 x 7 So the factors of 7 are: 1, 7

In fact, every number is evenly divisible by 1 and itself. Numbers like 7 that are ONLY divisible by 1 and themselves are called "prime" numbers.

Why is factoring important? We will use factoring to simplify fractions, to find common denominators of fractions, and to solve equations.

Practice Problems

Use PEMDAS and the math properties to solve the following expressions:

1. $3 + (2 + 3)^2 – [4(6 + 1)]$
2. $\sqrt{9} + 3 \times 7 – 4$
3. 5(x + 2)
4. 8 ÷ (4 x 2) x 3,473
5. 64 – [2(4 x 8)] + 8(3)
6. Factor 24 completely

Answer Key for Practice Problems

1. 0　　2. 20　　3. 5x + 10　　4. 3,473　　5. 24

6. 1, 2, 3, 4, 6, 8, 12, 24

Practice Problems Solved with Explanation

1. $3 + (2 + 3)^2 – [4(6 + 1)]$	Start with the inner parentheses
$3 + (5)^2 - [4(7)]$	Continue with the parentheses, and we can square 5 now.
$3 + 25 – 28$	Last, do the addition and subtraction from left to right.
0	
2. $\sqrt{9} + 3 \times 7 – 4$	Start with the square root of 9.
$3 + 21 – 4$	Multiply 3 x 7.
20	We are left with addition and subtraction.
3. 5(x + 2)	This is the distributive property. We leave the variable
5x + 10	next to the 5, indicating "5 times x" and multiply 5 x 2.
4. 8 ÷ (4 x 2) x 3,473	Start inside the parentheses, 4 x 2 = 8.
8 ÷ 8 x 3,473	Work the division and multiplication from left to right.
1 x 3,473	
3,473	
5. 64 – [2(4 x 8)] + 8(3)	Start inside the inner parentheses.
64 – [2(32)] + 24	Continue with the rest of the parentheses.
64 – 64 + 24	And then there is just addition and subtraction.
24	
6. 24 = 1 x 24 = 2 x 12 = 3 x 8 = 4 x 6	Starting with 1, list all the combinations of whole numbers that multiply together to get 24. Remember that 6 x 4 is the same as 4 x 6, so that combination does not need to be listed twice.
1, 2, 3, 4, 6, 8, 12, 24	