Understand the Story

Word problems connect math to real-world relevance. Math exists to solve our daily problems, but we often get too caught up in the "doing" of math to make the connection.

To solve a problem, we have to determine what facts we know and what facts we need to find. So first we need to just read and understand the problem. The first time you read it, ignore the numbers.

For example: "*Sally's house has 1,200 square feet of walls to be painted. She has already painted 800 square feet. How much still needs to be painted?*" Did you read that problem and begin to worry about how to calculate square feet? That might be natural, but it isn't what we need to solve.

Read the problem this way, ignoring the numbers: *"Sally's house has some walls to be painted. She has already painted some. How much still needs to be painted?"* That's the same word problem, using the word "some" to replace the quantities and details. Now it's simpler to know we need to solve.

There will be a list of keywords later, but you probably realize already that we just need to subtract the amount already painted from the total to find what's left to paint. The math is: 1,200 – 800 = 400. The answer is 400 square feet.

The more word problems you practice, the faster you will get. That's because you will begin to do some of the following steps at the same time. But if word problems are new or still challenging for you, try to follow these steps one at a time.

- Read the problem without the quantities to understand the story.
- Does the story fit a familiar pattern?
- Determine what detail you are asked to find; assign a variable.
- Organize the information you are given.
- Solve the problem.
- Ask yourself, "Does that answer make sense in the story?"

Familiar Patterns

Most word problems fit into one of three categories:

1) Chronological story with a "Start, an Event, and a Finish";

2) Relationship to apply; or

3) Formula to use or to create and use.

First, there is the chronological story. The story isn't always told in chronological order. You may be told the start and the finish and need to find out what event happened in the middle. You can recognize this type of word problem because it usually sounds most like a story, with a series of events, and a beginning, a middle and an end.

The example in the previous section with Sally painting her house is this type of problem. At the start, there were 1,200 square feet to paint. In the middle, an event occurred when she painted 800 square feet. So at the end of the story, there are still 400 square feet to paint.

You can organize this information by writing "Start," "Event," and "Finish." Then line up the information you are given in chronological order under these words. Once you do that, you can use your understanding of the story to fill in the math operations that make an equation. Then perform those operations.

Start		Event		Finish
1200 to paint		800 done		how much left
1200 to paint	-	800 done	=	how much left
1200	-	800	=	400

Here's another example: Joe needed to buy school supplies. He bought a pack of 12 pencils for $1.59, a backpack for $19.99, and three notebooks that were $2.99 each. He had $19.45 left. How much money did he start with? (If you didn't do this already, read it this way: Joe needed to buy school supplies. He bought some stuff. He had some money left. How much money did he start with?)

Start		Event		Finish
$X		Spent 1.59 + 19.99 + 3(2.99)		19.45
$X	-	Spent 30.55	=	19.45
$X	+	30.55	=	+30.55
$X			=	$50.00

We want to find how much money he started with. That's the unknown. The unknown can be represented by a variable, like X. So Joe's starting amount of money can be $X.

He bought some supplies, represented by the addition and multiplication listed as the event. At the finish, he had $19.45. Next, we put in the minus sign to show that he bought things (we all know that means subtract!). We also put in the equals sign to show the result of the event. Now we've got an equation, and we know already know how to solve equations. Solve the equation by adding 30.55 to both sides. We are left with $X = $50.00. So he started with $50.00.

The second common type of word problem is a relationship. These problems describe a proportional relationship which is likely to contain the word "per." Then you need to apply that relationship to another situation. For example, "Joe makes $15 per hour. How much does he earn in an 8-hour shift?"

For this type of problem, determine what two things have a relationship. In this case, it is dollars and hours. Write those things in fraction format, putting one in the numerator and one in the denominator. Then, keeping consistent with what is in the numerator and denominator, write the information you know about the relationship. Solve the relationship with cross-multiplication. Review Section 7, which has many other example of ratios and proportions and how to solve them with cross-multiplication.

$$\frac{dollars}{hour} = \frac{15}{1} = \frac{X}{8}$$

X = $120

The third main type of word problem involves a formula. It may be a known formula, like the volume of a cylinder, or it may be a formula that you have to derive from the story.

For a first example, consider a word problem that uses a known formula. "David's rectangular garden is 5' by 10.' What is the area of his garden in square feet?" There is an established formula for the area of a rectangle. You need to be familiar enough with common formulas to know it (or be able to look it up if allowed). It is Area = Base x Height.

First, write the formula. Next you need to organize the given data and substitute it into the formula, in this case "Area = 10' x 5'." The word problem may include a picture, or you may want to draw one yourself. Do what the formula tells you what to do, in this case multiply, and the answer is 50 square feet.

Area = Base x Height

Area = 10' x 5'

Area = 50 square feet

Here's another example, and in this case we need to create our own formula to solve the problem. "At the end of soccer tryouts, 20% of the children who tried out will be chosen for the team. If 15 children will be chosen for the team, how many tried out?" Read it without the quantities, "At the end of soccer tryouts, some of the children who tried out will be chosen for the team. If this number of children will be chosen, how many tried out?"

We know how many children were chosen, and we want to know how many children tried out. So, let's assign a variable that stands for the number of children who tried out (the unknown).

Let X = # of children who tried out

20% of X were chosen

15 were chosen

20% of X = 15
.20(X) = 15
÷.20 ÷.20
X = 75

After we assign the variable, we organize what we know. First, that 20% of X were chosen, and second, that 15 were chosen. That means that 20% of X is the same as 15. If something in math is the same as something else, you say they are equal and use an equal slgn. "Of" is an important keyword that means to multiply, and now we've got an equation. Divide both sides by .20 and we find that X = 75, so 75 children tried out for soccer.

Even when a story seems complicated, we can usually translate it directly from "English words" to "math words." Consider this example: "In five years, Jill will be four years less than two times her current age. How old is Jill?" Even if you aren't very familiar yet with keywords, you probably see the phrases "less than" and "two times." We know there's going to be some subtraction and some multiplication. We just need to get in in the right order.

The problem asks us for Jill's current age, so that's going to be the variable X.

Let X = Jill's age now

"In five years:" that's X + 5

"Jill will be:" any form of "is" "was" "will be" means equals sign

"Four less than two times current age:" that's 2X – 4

Put it together: X + 5 = 2X – 4
-X -X subtract X to combine like-terms
5 = X - 4
+4 + 4 add 4
9 = X so Jill is 9 years old

Notice the phrase "four less than some number." We have to write that as X – 4, not 4 – X. We know that when we subtract, the number we subtract FROM gets listed first. And even though we know this, it's an easy mistake because we want to write the equation exactly as we say it. We just need to be careful to write what we mean and not the first thing we hear.

Keywords

Below is a chart of some keywords and their likely meanings in word problems. No list like this is ever perfect or complete. There is no substitute for reading and understanding the problem. There is also no substitute for practice when it comes to solving word problems.

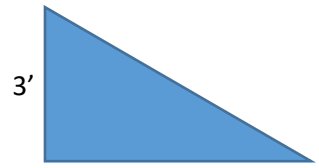
Consider these keywords a beginning point as you practice word problems.

<u>Addition</u> total combined altogether plus in all increase more	<u>Subtraction</u> minus difference left change fewer less decrease
<u>Multiplication</u> times of product	<u>Division</u> per out of share goes into
<u>Equals</u> is, are, was, were, will be gives yields results	

Practice Problems

1. Bob wants to take his family of six to the movies. Tickets for the afternoon show are $7 each, and tickets for the evening show are $11 each. How much will he save if they go to the afternoon show?

2. What is the area of the following triangle? (Area = $\frac{1}{2}$ base * height)



3. Jimmy collects vintage lunch boxes. He sold seven lunch boxes on an internet auction site. He bought four at a garage sale. If he started with 32 lunch boxes, how many does he have now?

4. Lorraine is a waitress who earns $8 per hour plus 15% of the food sales. If she worked a six-hour shift, and the food sales were $600, how much did she earn?

5. Part of $5,000 is invested at an annual rate of 5%, and the rest is invested at an annual rate of 3%. The total interest in one year is $210. How much was invested at 5% and how much was invested at 3%?

6. Alex drove 60 miles per hour on his way to a meeting for work. If it took him two and a half hours to get there, how many miles did he drive?

7. Shawna is baffled by the following problem: Think of a number. Double it. Add 6. Half it. Take away the number you started with. The answer will be 3. Can you write the equation that makes this always true?

8. Find four consecutive integers whose sum is four more than five times the first number.

Answer Key for Practice Problems

1. $24 2. 9 square feet 3. 29 lunch boxes 4. $138

5. $3000 at 5%; $2000 at 3% 6. 150 miles 7. $\frac{2x+6}{2} - x = 3$

8. 2, 3, 4, 5

Practice Problems Solved with Explanation

1. There are several ways to solve this. The point is to compare the two possibilities and find the difference between them. The unknown we are looking for is their savings, so let's assign:

X = $ savings

6 ($7) + X = 6 ($11)	Six $7 tickets + unknown savings will be the same as six $11 tickets.
$42 + X = $66	Multiply.
-$42 -$42	Subtract $42 from each side.
X = $24	The savings is $24.

2. We are given a formula and a picture, so we just need to put the given information into the formula.

Area = $\frac{1}{2}$ (base) (height)

= $\frac{1}{2}$ (6) (3)

= 9 Answers to word problems should get labeled in some way, 9 what? In this case it is 9 square feet.

3. This problem gives us a start and two events, although they are not given in that order, and we need to find the finish.

Start		Event		Event		Finish	
32		sell 7		buy 4		X	Put the information in chronological order.
32	-	7	+	4	=	X	Add the operations; sell means subtract; buy means add.
				29	=	X	Add and subtract; the answer is 29 lunch boxes.

4. Lorraine's income is a combination of a wage and her tips. The unknown we are looking for is her total earnings.

X = $ total earned	Assign the variable.
X = dollars (hours) + percent (sales)	Describe the equation with words.
X = $8 (6) + .15 ($600)	Substitute the numbers for the words.
X = $48 + $90	Multiply first.
X = $138	Add.

5. The $5,000 is divided into two amounts. One part is invested at 5%, one part at 3%. The formula for simple interest is Principal * Rate = Interest. We are asked to find both parts of the amount, so if we choose the amount invested at 5% for our variable:

X = $ invested at 5%	Assign the variable to stand for the first part.
5,000 – X = $ invested at 3%	"The rest" is 5,000 – the first part.
Principal (5%) + Principal (3%) = Interest	Describe the equation with words.
.05X + .03 (5000 – X) = 210	Substitute the numbers for words.
.05X + 150 - .03X = 210	Distribute .03 across (5000 – X)
.02X + 150 = 210	Combine like-terms: .05X = .03X = .02X.
.02X = 60	Subtract 150 from both sides.
X = 3,000	Divide both sides by .02.
5,000 – 3,000 = 2,000	We solved for $ invested at 5%, that's $3,000. The other part invested at 3% is $2,000.

6. This is a relationship problem, where the relationship we are given is between miles and hours.

$\frac{miles}{hours} = \frac{60}{1} = \frac{X}{2.5}$	Start by stating the relationship in fraction format. Keep the mile information in the numerator and hour info in the denominator.
X = 60 (2.5)	Cross-multiply.
X = 150 miles	Multiply.

7. We can add each step to the equation as we read it.

X = any number	Assign the variable
2X	"Double" it means multiply by 2.
2X + 6	Add 6.
$\frac{2X+6}{2}$	"Half it" means divide by 2, and we need to divide everything by 2, so everything we have so far is the numerator.
$\frac{2X+6}{2}$ – X	"Take away the number you started with" means subtract X.
$\frac{2X+6}{2}$ – X = 3	"The answer will be 3" means equals 3.

$2(\frac{2X+6}{2} - X) = 2(3)$	We weren't asked to solve, but we can if we multiply both sides by 2.
2X + 6 – 2X = 6	Combine like-terms.
6 = 6	We are left with something that is always true.

8. Sometimes people think that to find "four consecutive integers" means that the equation will have four variables. However, it's only necessary to assign X as the first integer, and then we can describe the others using X.

X = first integer	Assign X as the first integer.
X + 1 = second integer	That means the second integer is 1 more than X, and so on.
X + 2 = third integer	
X + 3 = fourth integer	
X + X + 1 + X + 2 + X + 3 = 4 + 5X	The sum of the number (add them together) is (equals) four more than (4 +) five times the first (5X)
4X + 6 = 4 + 5X	Combine like-terms
-4X -4X	Subtract 4X from both sides.
6 = 4 + X	
-4 -4	Subtract 4 from both sides.
2 = X	The first integer is 2
2, 3, 4, 5	Find the rest by going back to our original assignments.