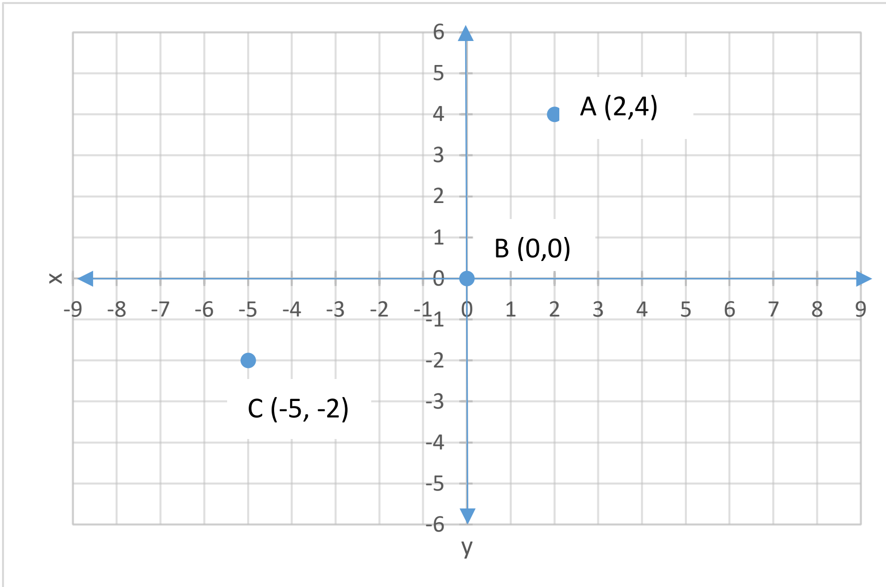
Graphing Points

Graphs show the location of a point and the properties of lines. The horizontal and vertical axes (lines) x and y on a graph are really just two number lines that intersect at a 90° angle.



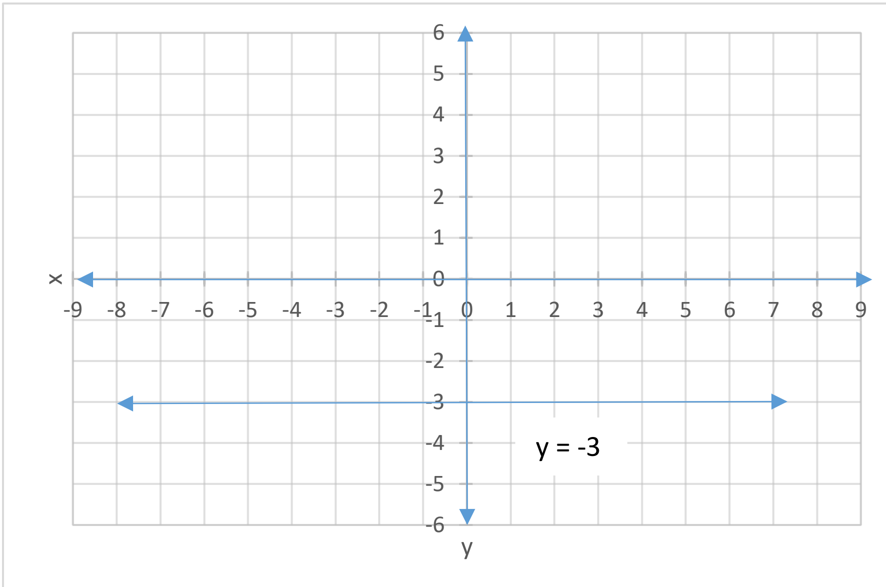
The "x-axis" is the horizontal number line through the center and the "y-axis" is the center vertical line. On the x-axis, the values get larger going to the right, and they get smaller going to the left. On the y-axis, the values get larger going up, and they get smaller going down.

Points on the graph are named with coordinates, a pair of numbers listed in parentheses. The coordinates called an "ordered pair," because the numbers are listed in order, x then y, in the format (x, y). So, the coordinates for point A on the above graph are (2, 4). Point A is the intersection where x = 2 (on the horizontal axis) and y = 4 (on the vertical axis).

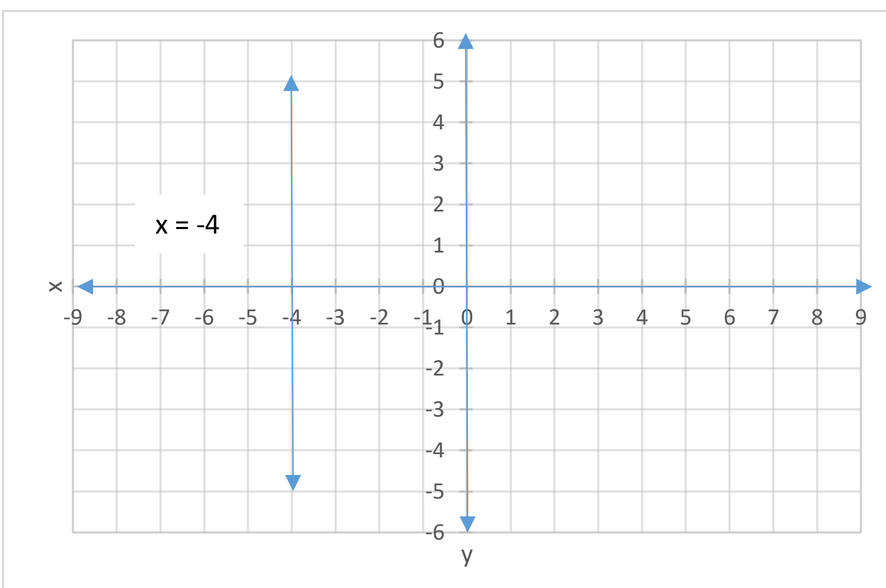
Point B has the coordinates (0, 0). It is the point where x = 0 and y = 0, which means it is at the "origin" (center) of the graph. Point C is (-5, -2), and it is an example of a point where both x and y are negative.

Graphing Lines

In this section we will only graph straight lines, or "linear" equations. The variables in linear equations do not have any exponents higher than one. Section 20 graphs non-linear equations that are curved.

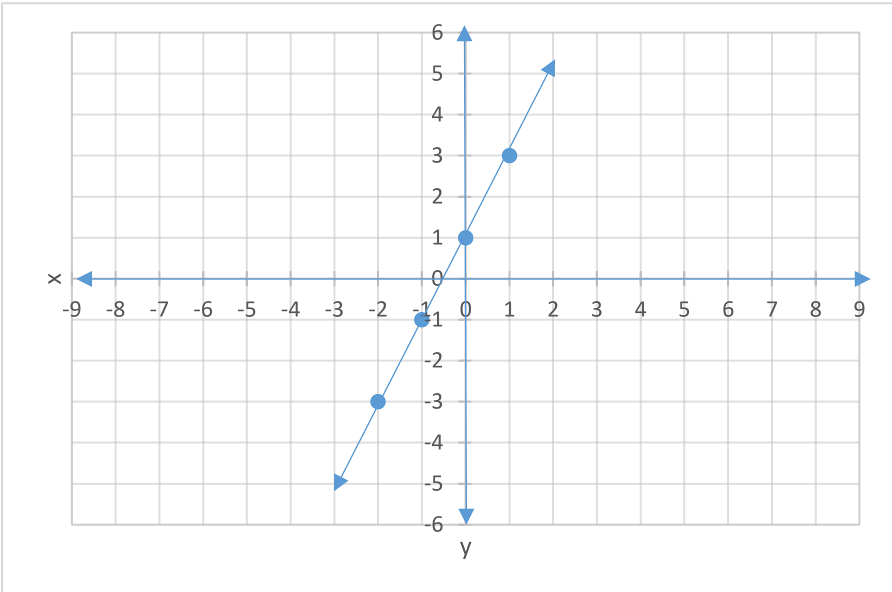


The equation for the line on the above graph is y = -3. For every point on that line, y is -3. Notice that the line crosses the y-axis at -3, so we say that its "y-intercept" is -3. This line does not ever cross the x-axis, so it does not have an "x-intercept." The graph below shows the line x = -4. This line crosses the x-axis at -4, its "x-intercept."



Slope

Slope describes how steep a line is and whether it slants in a positive or negative direction. The definition of slope is $\frac{rise}{run}$, where rise is the change in y values and run is the change in x values. When two points on a line are known, we can find the slope (or m) = $\frac{y1-y2}{x1-x2}$. Here is an example that shows both methods of determining slope.



The graph above shows a line that contains points (0, 1) and (1, 3). We can use the $\frac{rise}{run}$ definition of slope, and start at point (0, 1). From that point, the line rises two units (from 1 to 3 along the y-axis), and runs one unit (from 0 to 1 along the x-axis). That gives us $\frac{rise}{run} = \frac{2}{1}$. Simplified the slope = 2.

The formula $\frac{y1-y2}{x1-x2}$ measure the same thing. In this case y_1 means the y value of the first point, and y_2 means the y value of the second point, and likewise for x_1 and x_2. Using the coordinates from points (0, 1) and (1, 3) we get $\frac{y1-y2}{x1-x2} = \frac{1-3}{0-1} = \frac{-2}{-1} = 2$. We would get the same slope if we used (1, 3) as our first point and (0, 1) as our second point. It is not important which point is (x_1, y_1), but it is important that you consistently start both numerator and denominator with the coordinates of the same point.

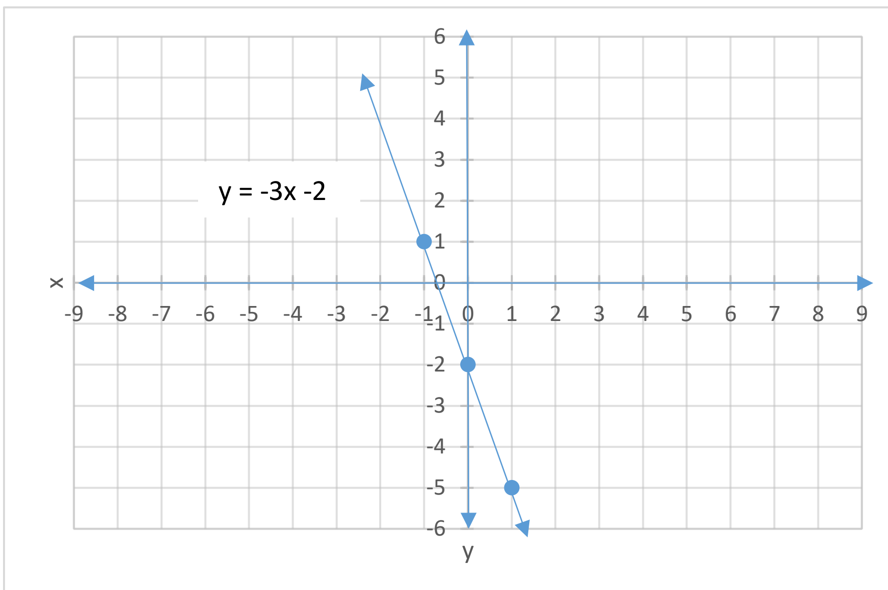
So, regardless of which method is used, we determine that slope of this line is 2. That is a positive slope, which means that the line rises as it moves left to right. A negative slope falls as it moves left to right.

There are two special cases of slope: horizontal lines (such as our previous example of y = -3) and vertical lines (such as our example of x = -4). Vertical lines have no “run,” which means the slope fraction would be divided by 0. Remember, we can’t divide by zero, and therefore, vertical lines have “undefined” slope. Horizontal lines have no “rise,” so the numerator of the slope fraction = 0. Horizontal lines have a slope equal to 0.

Graphing Lines with Slope-Intercept Form

The slope-intercept form is used when we know the slope of a line and its y-intercept. The standard format of the equation is y = mx + b. In this equation, x and y stand for values of x and y on the graph. Slope is represented by "m," and "b" represents the y-intercept.

For example, y = -3x – 2 is in the standard form. Slope in this example is -3. The y-intercept is -2. To graph the line, start by finding the point (0, -2) which is where the line will cross the y-axis. Then find another point by using the slope. In $\frac{rise}{run}$ format, the slope -3 is $\frac{-3}{1}$. That means that, starting at any point, we can find another point by going down three units and to the right one unit. We go down in this example because the slope is negative (-3).



The first point is plotted at (0, -2). From there, going down three units takes us to y = -5, and one unit to the right means that x = 1. So the next point is (1, -5). We could also move in the other directions since the slope can also be written as $\frac{3}{-1}$, so we rise three units (to y = 1) and run to the left (to x = -1). Drawing a line that connects our two (or more) points gives us the line y = -3x – 2. Every point on this line will give a true result when using its coordinates for x and y. Also, notice that the negative slope means that the slant of the line falls as it moves from left to right.

An equation doesn't have to start in the format of y = mx + b; if we solve the equation for y and rearrange the terms, we can still use this method to graph an equation. For example, take the equation 2x + 3y = 6 and solve for y.

$2x + 3y = 6$

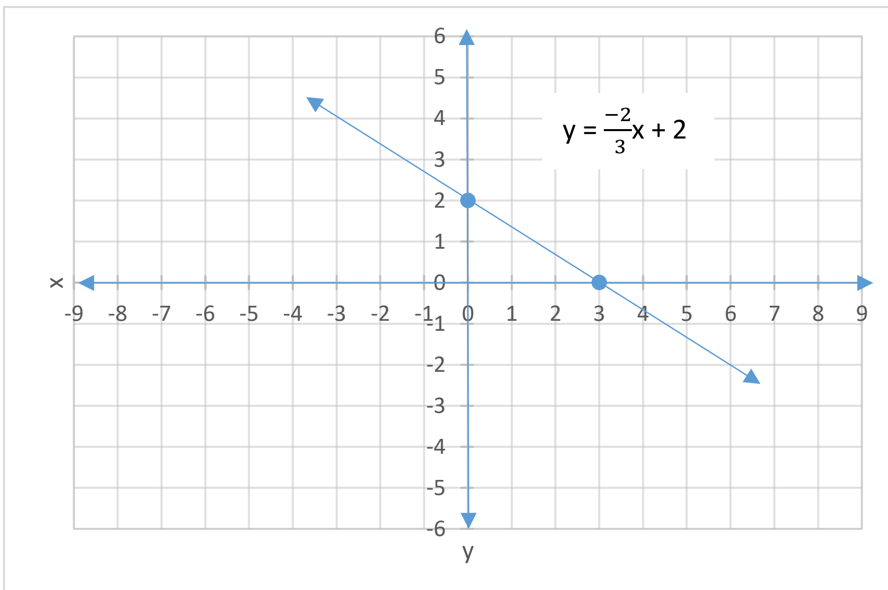
$\underline{-2x \quad\quad -2x}$ Subtract 2x from both sides to get the y-term by itself on the left.

$3y = -2x + 6$

$\underline{\div 3 \quad \div 3}$ Divide both sides by 3; remember to divide all terms by 3.

$y = \frac{-2}{3}x + 2$

So after solving for y, the equation is in the format y = mx + b. The slope (m) is $\frac{-2}{3}$. The y-intercept is 2. The line is graphed below. Start by plotting the y-intercept. The line will cross the y-axis at the value 2, which will be point (0, 2). The slope is negative, so "rise" is going down 2 units, and then the "run" will be moving to the right 3 units. That takes us to the point (3, 0). Draw the line that connects that point, which is the line $y = \frac{-2}{3}x + 2$.



Graphing Lines with Point-Slope Form

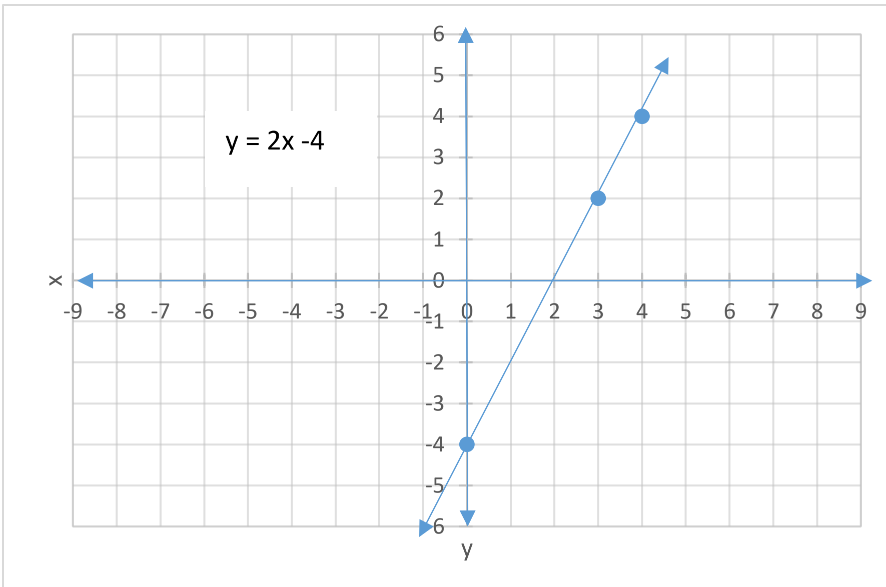
With any known point on a line and the slope, we can graph the line and we can determine the equation of the line. The standard form of this method is $y - y_1 = m(x - x_1)$. In this equation, x and y stand for values of x and y. The slope is represented by m. The x_1 and y_1 values represent the values of (x, y) from the point that we know.

For example, we know point (3, 2) is on a line where the slope is 2. To graph the line, we don't have to know the equation. We could just find point (3, 2), then find another point by "rising" two units and "running" one unit (since the slope is 2).

To find the equation of the line, use the formula $y - y_1 = m(x - x_1)$ and substitute the values that we know: the coordinates of point (x_1, y_1) and the slope, m.

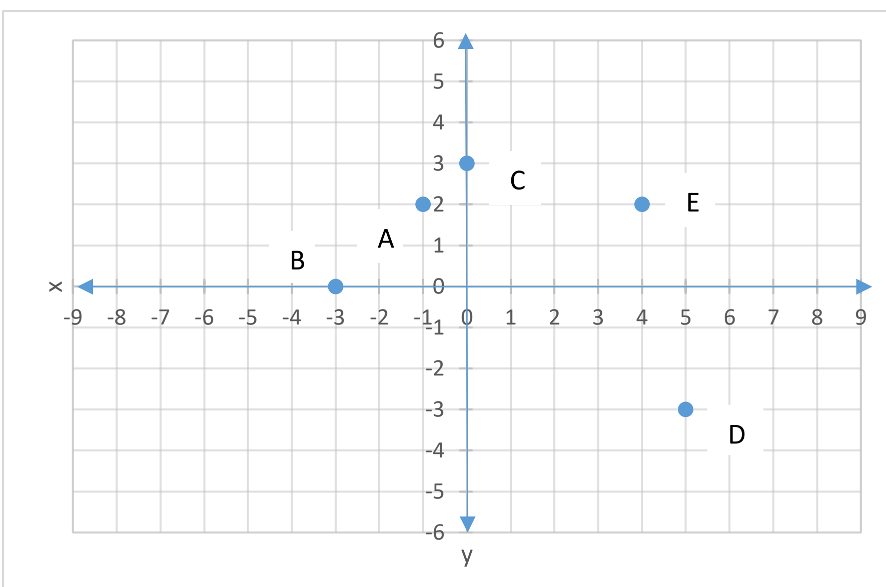
$y - y_1 = m(x - x_1)$
$y - 2 = 2(x - 3)$ Substitute $m = 2$; $x_1 = 3$; $y_1 = 2$
$y - 2 = 2x - 6$ Distribute the 2 across the $(x - 3)$
$\underline{\ +2 \qquad +2\ }$ Add 2 to both sides
$y \quad = 2x - 4$

And now draw the graph:



Practice Problems

1. Determine the coordinates of the following five points.

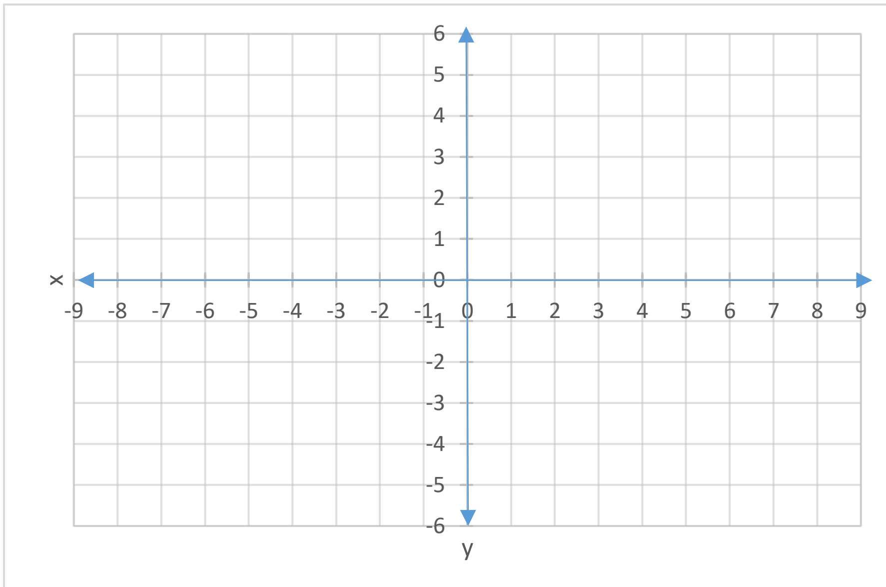


2. Find the slope of the lines that connect the following points:

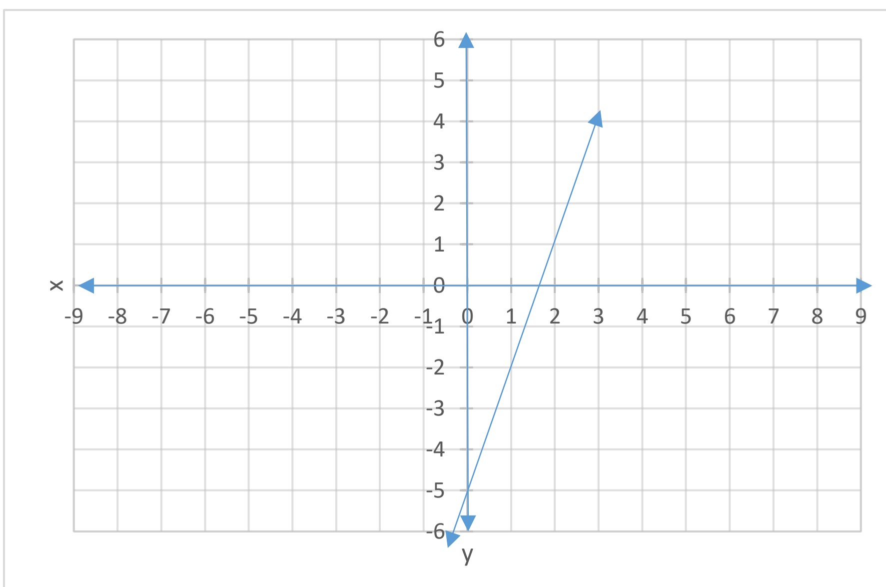
   a) (4, 2) and (1, 0)   b) (-2, 1) and (-4, 1)   c) (-3, -4) and (-1, 0)

3. Solve the equations in y = mx + b format and graph.

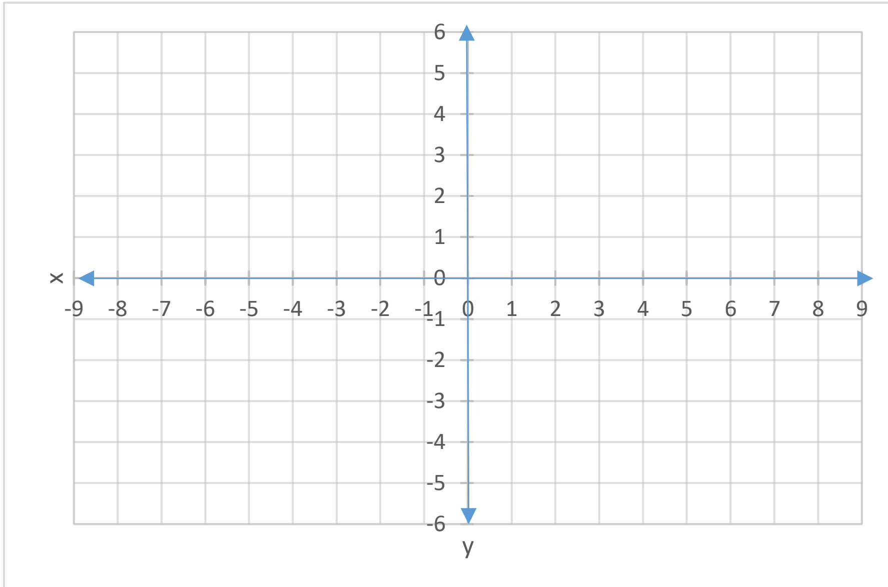
   a) 2y -2 = 3x   b) 6x + 3y = -9



4. What is the equation of the following line:



5. Find the equation of the line determined by point (2, 4) and slope $\frac{-1}{2}$ and graph the line. Determine whether the points (5,3) or (-2, 6) are on the line.

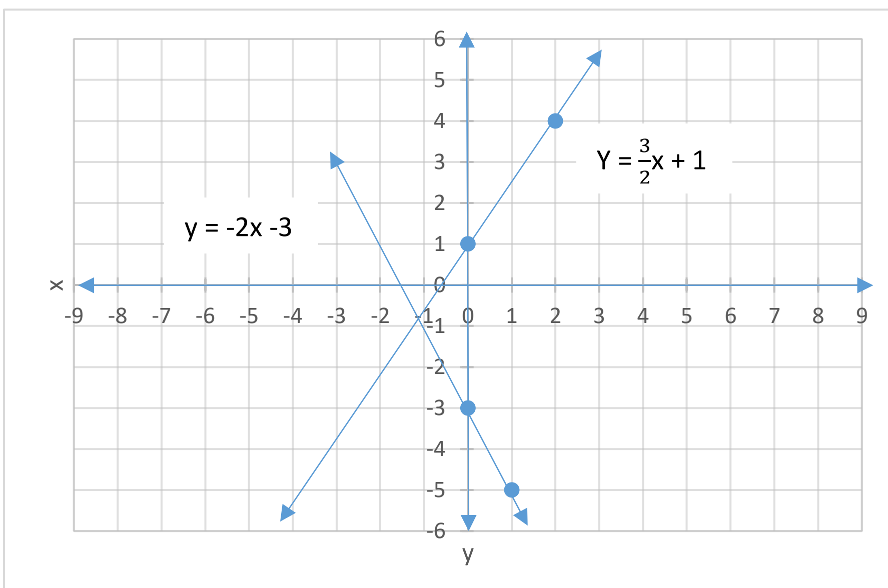


Answer Key for Practice Problems

1. a) (-1, 2) b) (-3, 0) c) (0, 3) d) (5, -3) e) (4, 2)

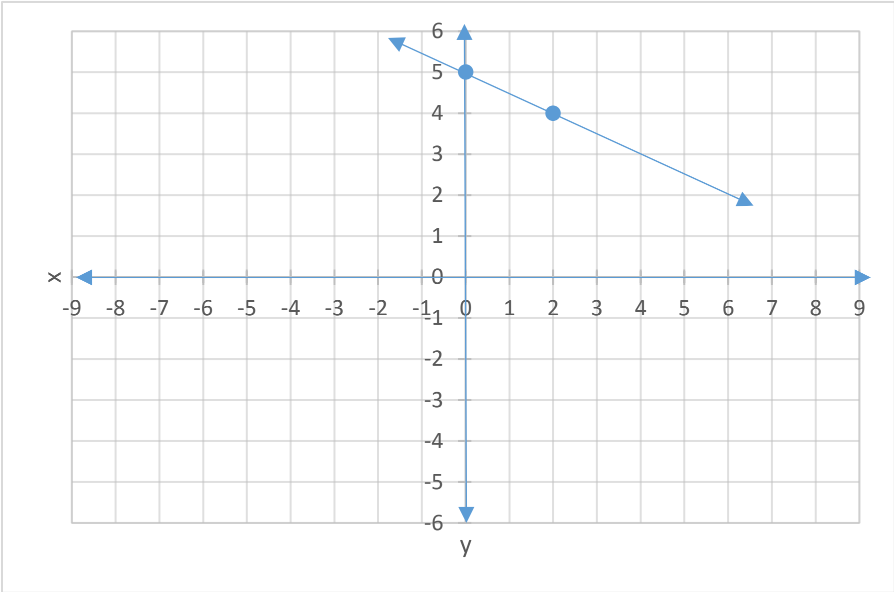
2. a) $\frac{2}{3}$ b) 0 c) 2

3. a) $y = \frac{3}{2}x + 1$ b) y = -2x -3

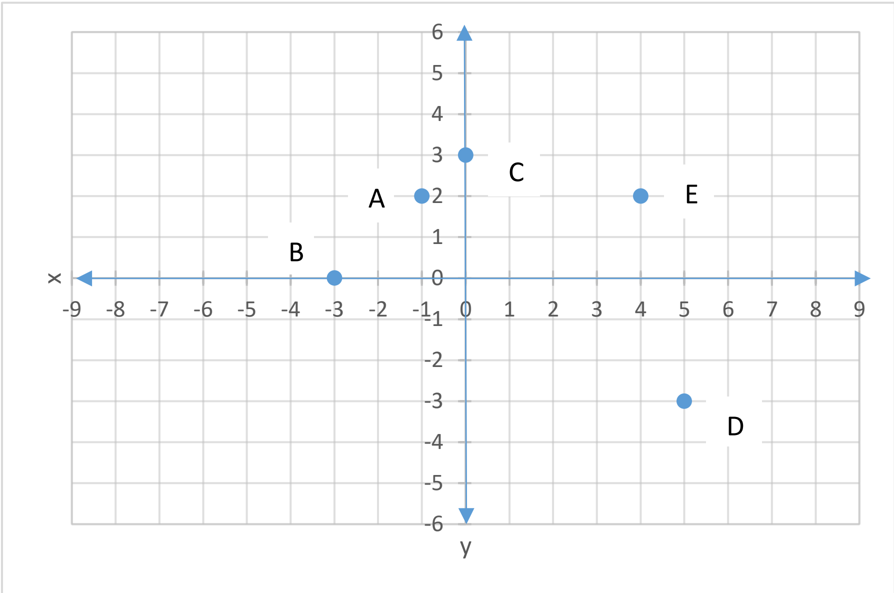


4. y = 3x -5

5. y = $\frac{-1}{2}$x +5; (5,3) is not on the line; (-2, 6) is on the line



Practice Problems Solved with Explanation



1. a) Point A lines up with the x-axis (horizontal axis) at -1, and it lines up with the y-axis (vertical axis) at 2. The coordinates are listed as (x, y), so Point A is (-1, 2).

1. b) Point B is on the x-axis at -3, and lines up with the y-axis at 0. In fact, remember that all points on the x-axis have a y-value of 0. The coordinates for Point B are (-3, 0).

1. c) Point C lines up with the x-axis at 0, and is on the y-axis at 3. Its coordinates are (0, 3).

1. d) Point D lines up with the x-axis at 5 and with the y-axis at -3, so its coordinates are (5, -3).

1. e) Point E lines up with the x-axis at 4 and with the y-axis at 2, so its coordinates are (4, 2).

2. a) The equation to find slope is $\frac{y1-y2}{x1-x2}$. We can call Point #1 (4, 2) and Point #2 (1, 0). So, for Point #1, (x_1, y_1) means x_1 = 4 (the x-value for Point #1), and y_1 = 2 (the y-value for Point #1). Likewise, x_2 = 1 and y_2 = 0. When we put these values into the formula, we get: $\frac{2-0}{4-1} = \frac{2}{3}$.

2. b) We can use (-2, 1) as Point 1, and (-4, 1) as Point 2. Remember, it doesn't matter which point is named #1 and #2, it only matters to consistently use one point first and the other point second in the formula. Putting the values into the formula gives us $\frac{1-1}{-2--4} = \frac{0}{2} = 0$. These two points have the same y-value, which means the line joining them will be a horizontal line, which has a slope of 0.

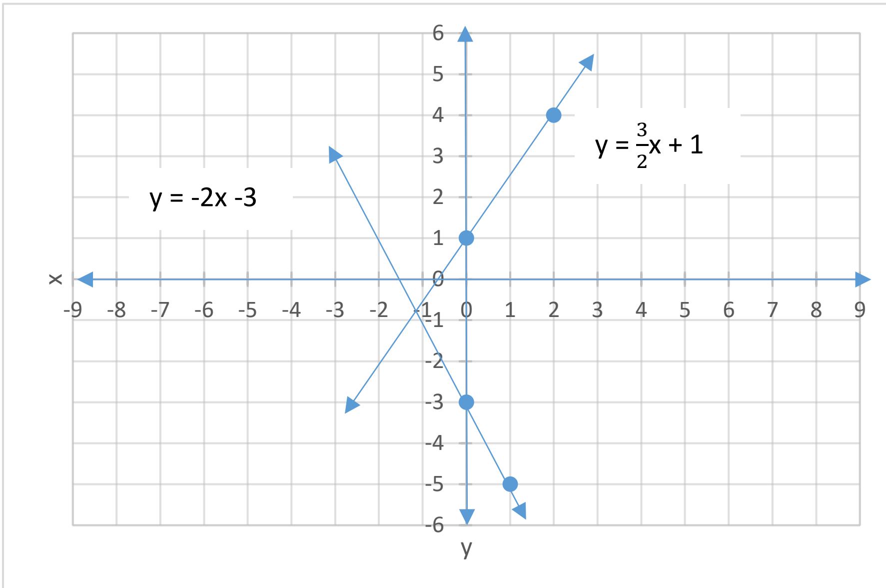
2. c) Using (-3, -4) as Point 1 and (-1, 0) as Point 2, the slope is $\frac{y1-y2}{x1-x2} = \frac{-4-0}{-3--1} = \frac{-4}{-2} = 2$.

3. a) 2y – 2 = 3x
+2 +2 Add 2 to both sides
2y = 3x + 2
÷2 ÷2 Divide both sides by 2. Don't forget to divide both terms on the right by 2.
$y = \frac{3}{2}x + 1$

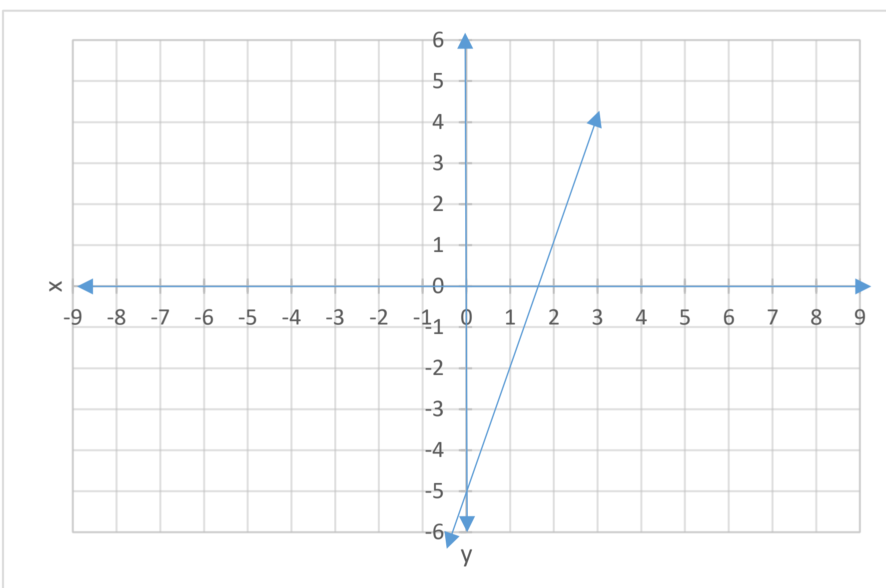
Now that the equation is in y = mx + b format, we can tell that the y-intercept (b) is 1 and the slope (m) is $\frac{3}{2}$. To draw the graph, start on the y-axis at 1, which is point (0, 1). The next point will have a rise (change in y) of 3 and a run (change in x) of 2. That means the next point will be (2, 4). The final step is to draw the line that joins those two points.

3. b) 6x + 3y = -9
-6x -6x Subtract 6x from both sides
3y = -6x -9
÷ 3 ÷3 Divide both sides (and all terms!) by 3
y = -2x -3

The equation is in y = mx + b format. The y-intercept (b) is -3, so the line will cross the y-axis at -3. That intercept is the point (0, -3). The slope (m) is -2. A negative slope means that the "rise" will actually be a decrease as the lines moves to the right. A negative rise, therefore, of -2 and a run of 1 means that the second point will be at (1, -5).



4. The line crosses the y-axis at -5, so the y-intercept (b) is -5. From that point, the line rises 3 units as it runs (to the right) one unit. That means the slope (m) is $\frac{3}{1}$ or 3. So for the format y = mx + b, we substitute those values and get y = 3x -5.



5. To find the equation of this line, we can use the point-slope form, which is $y - y_1 = m(x - x_1)$. Using the given slope (m) of $\frac{-1}{2}$ and the given point (x_1, y_1) of (2, 4), we first substitute those values into the equation:

$y - y_1 = m(x - x_1)$

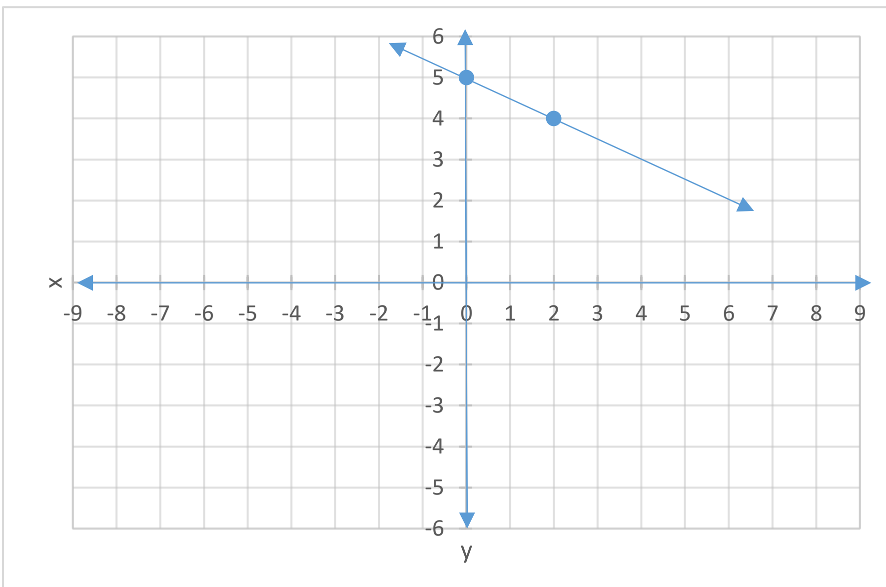
$y - 4 = \frac{-1}{2}(x - 2)$ Substitute in each value from the given point and given slope.

$y - 4 = \frac{-1}{2}x + 1$ Distribute the $\frac{-1}{2}$ across the contents of the parentheses, $(x - 2)$.

$+4 \quad +4$ Add 4 to both sides.

$y = \frac{-1}{2}x + 5$

We now know the equation for the line, and it is in point-intercept format, or y = mx + b. The y-intercept (b) is 5, which is the point (0, 5). The slope is $\frac{-1}{2}$. That means that the next line will fall (a negative rise) one unit while it runs (to the right) two units. That makes another point on the line (2, 4). The graph is below.



To determine whether a point is on a line, substitute the point's coordinates for x and y in the equation. If the result is true, the point is on the line.

$y = \frac{-1}{2}x + 5$

$3 = \frac{-1}{2}(5) + 5$ Substitute 5 for x, and 3 for y for point (5, 3)

$3 = \frac{-5}{2} + 5$ Multiply.

$3 = \frac{-5}{2} + \frac{10}{2}$ Convert 5 to a fraction with 2 as denominator.

$3 = \frac{5}{2}$ Add the fractions and convert to a mixed number.

$3 = 2\frac{1}{2}$ which is not true; so point (5,3) is not on this line

$y = \frac{-1}{2}x + 5$

$6 = \frac{-1}{2}(-2) + 5$ Substitute -2 for x, and 6 for y for point (-2, 6)

$6 = 1 + 5$ Multiply

$6 = 6$ Add. The result is true, so the point (-2, 6) is on this line.