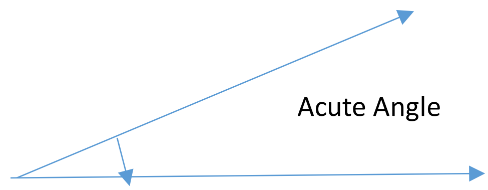
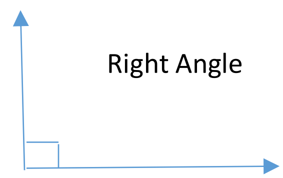
Triangles and Their Angles

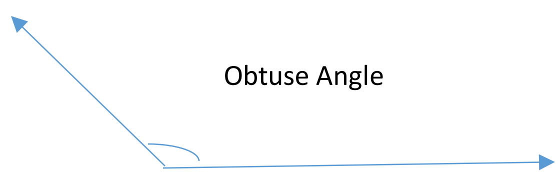
Triangles have three sides. They also have three angles. An angle is the measurement, usually given in degrees, between two lines that meet at a point (called the vertex). An angle that is less than 90°, like the one below, is called an “acute” angle.



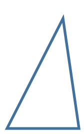
An angle that is exactly 90° is called a “right” angle, and is usually drawn with a square shape in the inner part of the angle.

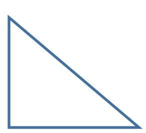


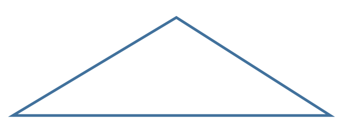
The third type of angle is “obtuse,” and it is an angle that is more than 90°.



When triangles are described based on their angles, they are called either “acute triangles,” “right triangles,” or “obtuse triangles.”


Acute Triangle

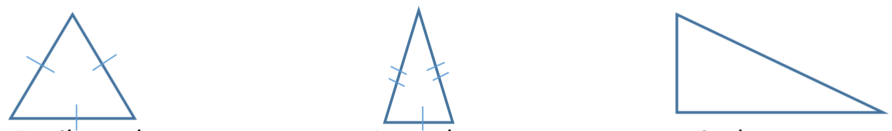

Right Triangle


Obtuse Triangle

All the angles in an acute triangle are less than 90°. One angle in a right triangle is exactly 90°, and one angle in an obtuse triangle is greater than 90°. In all triangles, the measure of all angles adds up to 180°.

Triangles and Their Sides

Triangles can also be described by the length of their sides. When all three sides of a triangle have the same length, the triangle is called "equilateral." When just two sides of a triangle have the same length, the triangle is called "isosceles." And when all three sides of a triangle have a different length, it is called a "scalene" triangle.

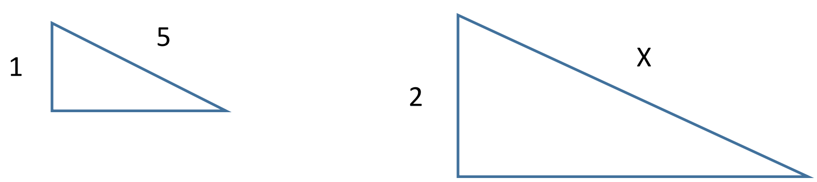


The small lines through the sides of the triangles are a math "shorthand" method to indicate equal lengths. Any matching number of lines indicates those lengths are equal. In the equilateral triangle above, all sides are equal, so there are matching lines through all three sides. In the isosceles triangle, the matching set of two lines through the upright sides indicates that the length of those sides is equal. The base of the isosceles triangle is not equal to the other sides, so it has a different number of lines through it.

Congruent and Similar Triangles

When two triangles have sides that are the same length and angles that are the same measurement, they are called "congruent" triangles. Congruent triangles are exactly the same size in every aspect, although they could be drawn as mirror images or rotated.

Similar triangles also have congruent angles, that is, their corresponding angles have the same measurements. However, the lengths of the sides of similar triangles are proportional rather than identical. As an example, look at these two triangles:



If we know that these two triangles are similar, we know that the lengths of their corresponding sides are proportional. In this case it is a simple proportion: the second triangle is twice as long as the first triangle. We can use proportions (see Section 7 for review) to create an equation to find X. Compare the corresponding sides and their measurements, in this example:

$$\frac{shortest\ side}{longest\ side} = \frac{1}{5} = \frac{2}{X}$$

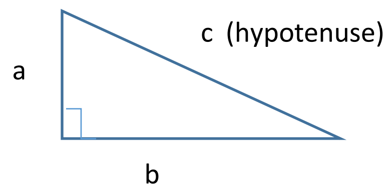
$$10 = X$$

Using cross-multiplication, we find that side X = 10.

Pythagorean Equation

The Pythagorean Equation only applies to right triangles. The equation lets us find the length of any third side of a right triangle when we already know the length of two sides.

The equation is: $a^2 + b^2 = c^2$, where a and b are the shorter sides of a right triangle, and c is its hypotenuse. The hypotenuse is the longest side of a right triangle; it is always opposite the right angle. For example:



In this example, if a = 3 and b = 4, we can find the length of side c.

$a^2 + b^2 = c^2$

$(3)^2 + (4)^2 = c^2$ Remember that $(3)^2$ means multiply 3 by itself: 3 x 3 = 9

$9 + 16 = c^2$

$25 = c^2$ Take the square root of both sides; $\sqrt{25} = 5$

$5 = c$

The equation can also be used when we know the hypotenuse and one side. If a = 5 and c = 13, find b:

$a^2 + b^2 = c^2$

$(5)^2 + b^2 = (13)^2$

$25 + b^2 = 169$

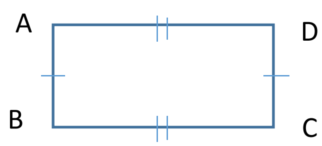
$b^2 = 144$

$b = 12$

These two examples both use common "Pythagorean Triples," which are whole numbers that work in the Pythagorean relationship. It is useful to recognize these common patterns because they often appear in homework and test questions. These triples work in their proportions too. For example, because (3, 4, 5) can be the sides of a right triangle, (6, 8, 10) and (9, 12, and 15) can also be the sides of a right triangle.

Rectangles and Squares

A quadrilateral has four sides. Rectangles are special quadrilaterals, where the length of their opposite sides are equal and all their angles are right angles.

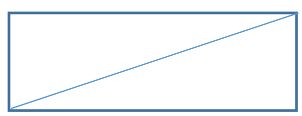


The rectangle above is called "Rectangle ABCD," named after its four points. Sides AD and BC are the long sides, and by the definition of a rectangle they are equal in length. Sides AB and CD are the short sides; they are also equal in length.

A square is also a rectangle, but it is a special rectangle where all four sides are equal in length.

All four angles in a rectangle (or a square) are right angles which equal 90° each. That means that the total of the angles of a rectangle is 360°, or 90° x 4.

The total of the angles of a triangle is 180°. To visualize why, remember that every triangle is half of a quadrilateral. Half of 360° is 180°.

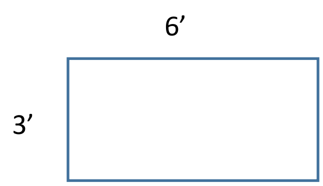


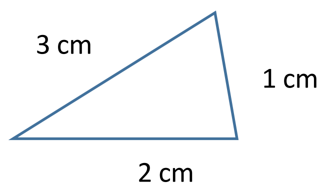
Perimeter

Perimeter is the distance around the outside of a shape. We can measure the perimeter of a square, rectangle, triangle, or any irregularly shaped object by adding together the lengths of its sides.

For example, the perimeter of the rectangle below is 3′ + 6′ + 3′ + 6′ = 18′. Since the shape is a rectangle, its opposite sides are equal. Because of that fact, we could also calculate the perimeter of the rectangle as 2(3′) + 2(6′). If units of measure are given, in this case feet, the answer should include the unit of measure. The perimeter of the rectangle is 18 feet.

The perimeter of the triangle is 3 cm + 2 cm + 1 cm = 6 cm. Therefore, the distance around the triangle is a total of 6 cm. We would use the perimeter measurement if, for example, we wanted to find the amount of fencing required to enclose these shapes.

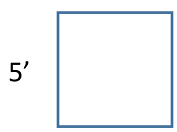




Area

Area is the total amount of space taken up inside a flat shape. There are many practical uses for determining the area of a space. For example, you may need to know the square feet of your driveway in order to buy enough black top; you may need to know the area of a shirt or sweater to know how much fabric will be needed.

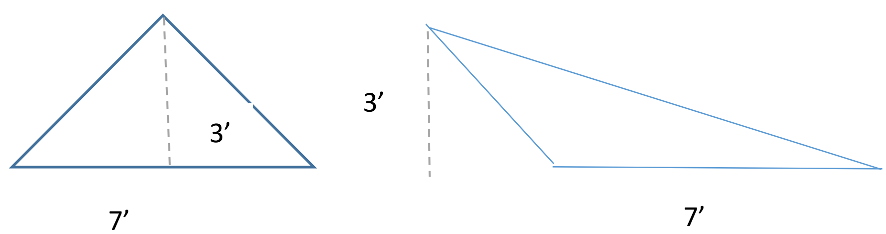
The formula for area of a rectangle is length x width. The result should be given in square units, for example square feet. "Length" and "width" can also be called "base" and "height." Both sets of names refer to the same measurements.



The figure above is a square with sides 5'. Since we know that in a square all four sides are equal, the area of the square is length x width = 5' x 5' = 25 square feet.

The formula for the area of a triangle is $\frac{1}{2}$ base x height. If you remember that a triangle is half of a quadrilateral, this formula makes perfect sense.

For example, the measurements of the triangle below are a height of 3' and a base of 7'. To calculate the area using the formula $\frac{1}{2}$ x base x height, we get $\frac{1}{2}$ (7') (3') = 10$\frac{1}{2}$ square feet.



Note that height doesn't have to be one of the sides of the triangle, it may be measured inside (or outside) the shape.

Practice Problems

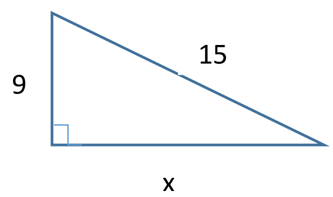
1. Determine whether the following sets of angles represent an acute triangle, right triangle, or obtuse triangle.
   a) 90°, 50°, 40° b) 120°, 30°, 30° c) 60°, 60°, 60°

2. Determine whether the following sets of side lengths represent an equilateral triangle, isosceles triangle, or scalene triangle.

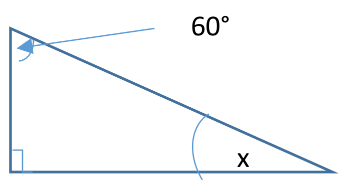
   a) 5", 5", 3" b) 3 cm, 4 cm, 5 cm c) 4', 4', 4'

3. Given that these two triangles are similar, find the value of x.

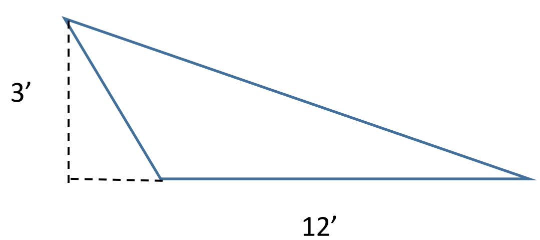
4. Find x.



5. Find the measure of angle x.

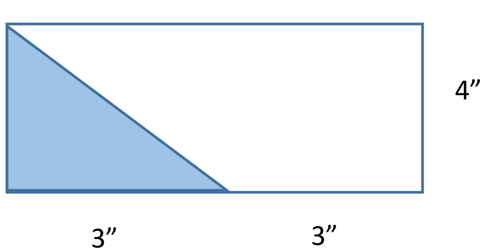


6. Find the area of the following triangle.

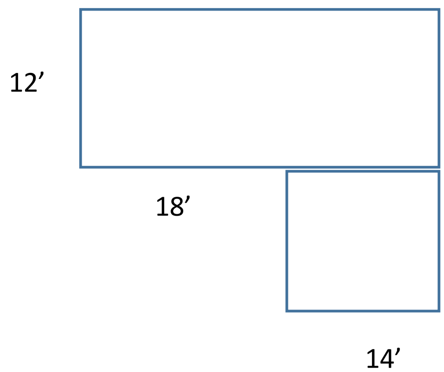


7. Joe wants to fence his garden, which is 50’ long and 20’ wide. How many feet of fencing does he need to buy?

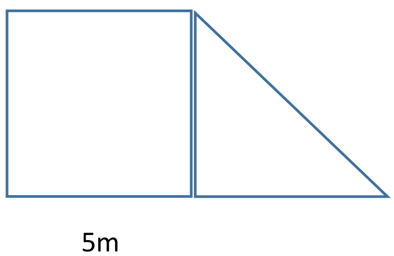
8. What is the area of the non-shaded part of this shape:



9. Find the total area of a room that has a 14’ square dining room connected to a living room with the following dimensions.



10. An isosceles right triangle is connected to a square with a side of 5m. What is the area of the resulting shape?



Answer Key for Practice Problems

1. a) right triangle b) obtuse triangle c) acute triangle

2. a) isosceles triangle b) scalene triangle c) equilateral triangle

3. 9 4. 12 5. 30° 6. 18 square feet 7. 140’ 8. 18 square inches

9. 580 square feet 10. 37.5 square meters

Practice Problems Solved with Explanation

1. a) One of the angles is 90°, which indicates a right angle. Any triangle with a right angle is a right triangle.

1. b) One of the angles is greater than 90°, which is an obtuse angle.

a. c) All the angles are less than 90°, which makes this an acute triangle.

2. a) Two sides are 5” and the third side is different, making this an isosceles triangle.

2. b) All three sides are different, which indicates a scalene triangle.

2. c) All three sides are equal, so this is an equilateral triangle.

3. We know that the triangles are similar, which means their sides are proportional. We can create a proportion equation. First state which sides are corresponding, then list the values for those sides.

$$\frac{side}{base} = \frac{2}{6} = \frac{3}{x}$$

18 = 2x Cross-multiply.

9 = x Divide both sides by 2.

4. This is a right triangle. We know that because there is a square box in the lower left angle that indicates it is a right angle. When we know two sides of a right triangle, we can use the Pythagorean Equation.

$a^2 + b^2 = c^2$ This is the Pythagorean equation. a and b are the sides; c is the hypotenuse.
$9^2 + x^2 = 15^2$ Remember that to square a number, multiply it by itself.
$81 + x^2 = 225$
$x^2 = 144$ Subtract 81 from both sides.
$x = 12$ Take the square root of both sides (what number multiplied by itself = 144? 12)

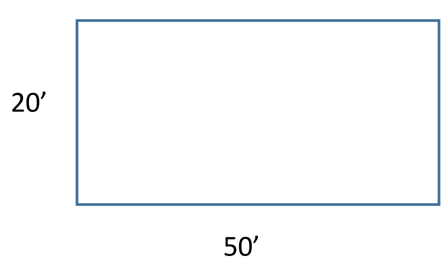
5. The angles in a triangle always add up to 180°. We know that one of these angles is a right angle because it is marked with a square. Right angles are 90°. Another angle is marked 60°. Create an equation that shows that the three angles add up to 180.

$90° + 60° + x = 180°$
$x = 30°$ Subtract 90 and 60 from 180.

6. The formula for the area of a triangle is $\frac{1}{2}$ base x height. The base of this triangle is labeled 12’. The height of this triangle s 3’, which is shown outside the triangle with the dotted lines.

$\frac{1}{2}$ base x height
$\frac{1}{2}$(12) x (3) Substitute the actual measures for the base and height.
18 square feet. The measurements are given in feet, so the answer is labeled as square feet.

7. We know that Joe’s garden is 50’ long and 20’ wide, which must be a rectangle that looks like this:



We want to know how much fence will go around the garden, which is the perimeter. To find the perimeter, add the lengths of all sides: 20’ + 50’ + 20’ + 50’ = 140’.

8. In order to find the area of the non-shaded part of the shape, we will need to find the area of the entire shape (the rectangle) and then subtract from that the area of the shaded triangle.

The rectangle is 4" tall, and a total of 6" wide. (There are two sections of 3" each, for a total of 6".) The area of the rectangle is base x height. That is 6" x 4", or 24 square inches.

The triangle is a right triangle. Although the angle is not marked, it is inside a rectangle and we know that the angles of rectangles are 90°. The base of the triangle is 3". The height is 4" (the same as the height of the rectangle). So the area of the triangle is $\frac{1}{2}$ base x height. That is $\frac{1}{2}$ (3) (4), or 6 square inches.

Finally, to find the area of the non-shaded part, subtract the triangle's area from the rectangle's area: 24 sq. in. – 6 sq. in. = 18 sq. in.

9. The total area of the room is the area of the rectangular part plus the area of the square part. Only part of the base of the rectangle is given, which is 18'. We know that the total base of the rectangle is that 18' plus the 14' that shares a border with the square. Therefore, the total base is 18' + 14' = 32'. The height of the rectangle is 12'. We use the formula base x height to find the area of the rectangle. 32' x 12' = 384 square feet.

The area of the square portion of the room is 14' x 14' = 196 square feet. Add these two portions of the shape together to get the total: 384 + 196 = 580 square feet.

10. It is important to read all the clues here. The right triangle is also an isosceles triangle, so the two shorter sides are also 5m. We can find the total area by adding together the areas of the two parts.

The area of the square is base x height. We know that a square has the same measurement for all of its sides, so base x height = 5 x 5 = 25 square meters.

The area of the triangle is $\frac{1}{2}$ base x height = $\frac{1}{2}$ (5)(5) = $12\frac{1}{2}$ square meters. Adding the two parts together, we get 25 + $12\frac{1}{2}$ = $37\frac{1}{2}$ square meters. It is fine to leave it in fraction form, or convert the fraction to a decimal and write this as 37.5 square meters.