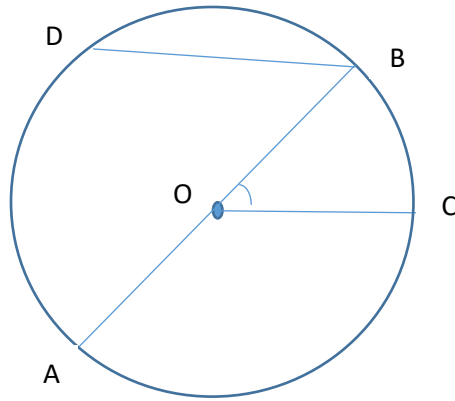## Parts of a Circle

A circle contains all the points equal in distance from a center point, or "origin." In the circle above, the origin is the point labeled O.

The diameter of a circle is a line from one side of the circle to the other side which passes through the origin. In this circle, it is the line AB.

The radius of a circle is a line from the origin to the side. One radius in this circle is the line OC. The radius is $\frac{1}{2}$ the length of the diameter.

Circumference is the measurement around the entire circle (perimeter).

An arc is a segment of the circumference. In this circle, one arc is from B to C along the circumference.

A central angle is an angle that begins at the origin of a circle, and whose legs (lines) are radii of the circle. There is a central angle marked with legs OB and OC. A complete circle has 360°, so a central angle is a portion of 360°.

A chord is a line that connects two points on the circumference, but does not necessarily go through the origin. In this diagram, the line BD is a chord.

Pi and Irrational Numbers

Pi, shown by the Greek symbol π, is the ratio of a circle's circumference to its diameter. All circles are proportional to each other, so that ratio is constant. Pi can be approximated in decimal format as 3.14, and pi can be approximated as the fraction $\frac{22}{7}$.

When the actual lengths of circumference and dimeter are measured with extreme accuracy, the result of calculating pi is a non-ending and non-repeating decimal. Hundreds, thousands, even millions of

decimal places can be calculated for pi, and the numbers will never end or form a repeating pattern. Pi is therefore an "irrational" number.

Rational numbers, by comparison, are numbers that DO end, and DO NOT repeat without a pattern. Rational numbers can be written as a fraction of two integers.

An integer is a number with no fractional parts. Integers include the counting numbers (1, 2, 3, …), zero (0), and the negative of the counting numbers (-1, -2, -3…).

For example, the fraction $\frac{1}{3}$ is a rational number. It is a fraction of two integers. The decimal equivalent of $\frac{1}{3}$ is .3333…, where the threes repeat forever. Even though this decimal repeats forever, there IS a pattern (all threes), so it fits the definition of a rational number.

Basic Circle Formulas

The radius is half the length of the diameter (r = $\frac{1}{2}$D). That, of course, means that the diameter is twice the length of the radius (D = 2r). In any problem where one measure is given, we can determine the other.

All circles have the same proportion of circumference to diameter; that is π (pi). π = $\frac{circumference}{diameter}$, or we can write that π = $\frac{c}{D}$. That means that circumference = π(diameter), which is written c = πD. The relationship also means that diameter = $\frac{circumference}{\pi}$, or D = $\frac{c}{\pi}$.

$$\pi = \frac{c}{D}$$

$$c = \pi D$$

$$D = \frac{c}{\pi}$$

If we know the circumference of a circle, we can find its diameter. For example, in a circle with a diameter of 7 meters, find the circumference:

   c = πD
   so c = π(7)
   Using 3.14 as an approximation of π, c = 3.14(7), so the circumference = 21.98 meters.

   Using $\frac{22}{7}$ as an approximation of π, c = $\frac{22}{7}$ x $\frac{7}{1}$ = 22 meters.

Notice that $\frac{22}{7}$ and 3.14 are <u>approximations</u> of the measurement of pi. When we need more accurate results, we can use as many decimal positions for pi as necessary to get the best results. In this section, we will use 3.14 as the value of pi.

If we know the diameter of a circle, we can find its circumference. For example, in a circle with a circumference of 8 inches, find the diameter:

D = $\frac{c}{\pi}$

So D = $\frac{8}{3.14}$ = 2.55 inches (rounded to the nearest hundredth).

The area of a circle = πr^2.   We read that as "pi r squared." It means that we square the measure of the radius (multiply it by itself), and multiply that result by π.

For example, find the area of a circle with a radius of 3 feet.

Area = πr^2
Area = 3.14(3^2)
Area = 3.14(9)
Area = 28.26 square feet

We can use the area formula to find the radius (or diameter) of the circle also. For example, find the diameter of a circle with an area = 72 square inches (and round to the nearest hundredth).

Area = πr^2
72 = (3.14)r^2
22.93 = r^2 Divide both sides by 3.14
4.79 = r   Find the square root of both sides. Radius = 4.79 inches.
Diameter = 2r, so diameter = 2(4.79). The diameter is 9.58 inches.

Volume

Volume is the amount of material that can fill a three-dimensional shape. Knowing how to find an object's volume tells how much a liquid a container can hold; how much water it takes to fill a bath tub; and how much wood is in a solid door.

There are similarities between the area formulas for two-dimensional (flat) shapes and their three-dimensional versions. For example, the area of a rectangle = length x width. The volume of a box = length x width x height.
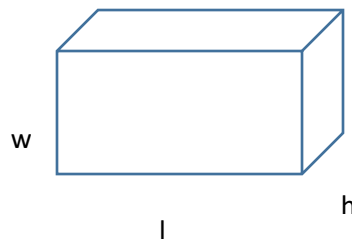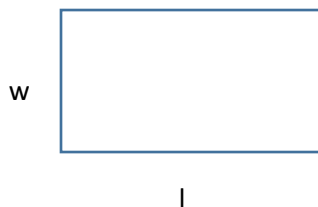

It really doesn't matter what dimension we label "height," "width," or "length." The box will have the same volume whether it is turned on its "side" or stood up on its "end."

The volume of a box that is 3" wide, 5" long and 2" high is calculated as:

Volume of box = lwh = 5" x 3" x 2" = 30 cubic inches.

We show the results of volume in cubic units, like this example where the answer is given in cubic inches. All the measurements must be in the same units before finding the volume.

A cube is a special box where all the dimensions are the same. In the cube below, all the sides are 4", so the volume is 4" x 4" x 4" or 64 cubic inches. This is the same as 4^3, "four cubed" or "four to the 3rd power." To cube a number, multiply three of that number together.

4"
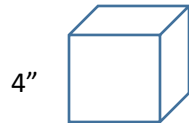
A cylinder has a circle for its base. The volume of a cylinder is $\pi r^2 H$. (That's the same as the area of a circle times the height of the cylinder.)
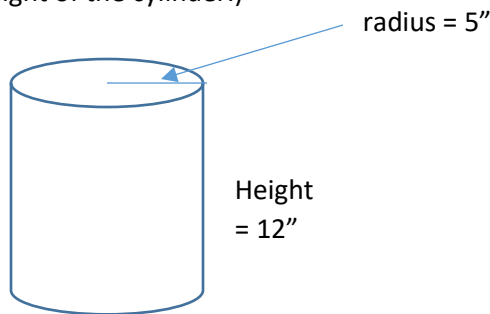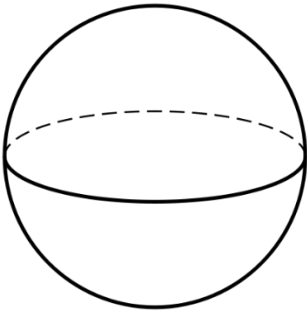
radius = 5"

Height = 12"

Radius = 5"; Height = 12:
$V = \pi r^2 h$
$= \pi(5^2)(12)$
$= 3.14(25)(12)$
$= 942$ cubic inches

A cone has a circle for a base and a tapering, triangle-shaped end. The volume of a cone is $\frac{1}{3}\pi r^2 H$. That means it has $\frac{1}{3}$ the volume as a cylinder with the same radius and height.
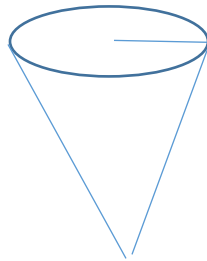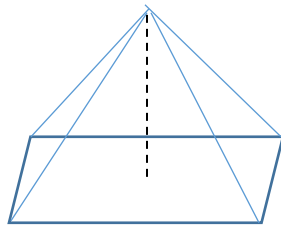
Radius = 5"
Height = 12"
Volume = $\frac{1}{3}\pi r^2 H$
$= \frac{1}{3}(3.14)(5^2)(12)$
$= 3.14(25)(4)$
$= 314$ cubic inches

A sphere is a three-dimensional circle, like a ball or a globe. The volume of a sphere $= \frac{4}{3}\pi r^3$.

Radius = 5"

Volume $= \frac{4}{3}\pi r^3$

$\quad = \frac{4}{3}(3.14)(5^3)$

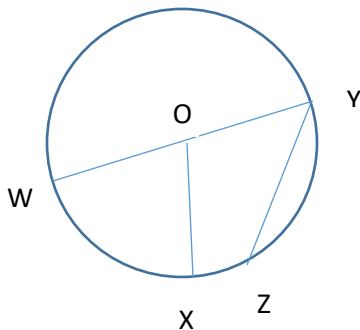$\quad = \frac{4}{3}(3.14)(125)$

$\quad = 523.3$ cubic inches (to nearest tenth)

A pyramid has a rectangle for a base and four tapering triangle-shaped sides. The volume of a pyramid = $\frac{length \ x \ width \ x \ height}{3}$.

Length = 6"

Width = 3"

Height = 8"

Volume $= \frac{lwh}{3}$

$\quad = \frac{6(3)(8)}{3}$

$\quad = 48$ cubic inches

Practice Problems

1. Match the line segments in this circle with its proper label.

a) OX   i) diameter
b) WY   ii) chord
c) YZ   iii) arc
d) WX   iv) radius

2. What is the circumference of a circle with a radius of 12" (to the nearest hundredth)?

3. What is the diameter of a circle with circumference of 16" (to the nearest tenth)?
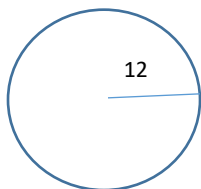
4. What is the area of a circle with radius of 5" (to nearest tenth)?

5. What is the radius of a circle with area of 200.96 square feet?

6. What is the volume of a cube with side of 1"?

7. A father is making a rectangular storage box for his son's building blocks. Each building block is 1 cubic inch, and the son has 500 blocks. If the base of the storage box is 5" by 10", how tall does it need to be to hold all of the blocks?

8. Mary's water bottle is a cylinder with a radius of 1.25" and a height of 6". How many ounces of water will it hold if each ounce of water fills 1.8 cubic inches (to the nearest ounce)?

9. Lucky Licks Ice Cream Shop buys ice cream cones with a diameter of 8 centimeters and a height of 15 centimeters. If ice cream is tightly packed into the cone (but not overflowing), how many cubic centimeters of ice cream does it hold?

10. How many cubic inches of air are needed to fill a ball with a circumference of 10", to the nearest tenth?

11. What is the volume of a pyramid with a length of 9cm, width of 12 cm, and height of 6cm?

Answer Key for Practice Problems

1. a) radius   b) diameter   c) chord   d) arc

2. 75.36"   3. 5.1"   4. 78.5 square inches   5. 8'   6. 1 cubic inch

7. 10"   8. 16 oz   9. 251.2 cubic cm   10. 16.8 cu in   11. 216 cu cm

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Practice Problems Solved with Explanations

1. a) The line OX is from the center to the side, making it a radius.
    b) The line WY is from side to side through the center, so it is the diameter.
    c) YZ is from one side to another, so it is a chord.
    d) XZ is a portion of the circumference, making it an arc.

2. It is always a good idea to draw a picture and label it with the information from the problem. The next step is to identify what you are asked to find: we want to find the circumference. Then write the formula for circumference, which is $c = \pi D$.

12

Fill in the known information, $c = \pi[2(12)]$. Remember diameter = 2(radius).

$$c = 3.14(24)$$
$$c = 75.36" \quad \text{Unit of measure is inches.}$$

3. First draw a picture of the circle. We are looking for the diameter. We know the circumference is 16". The formula for finding diameter of a circle is D = $\frac{circumference}{\pi}$.
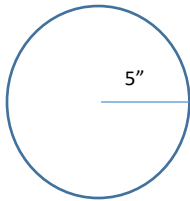
$$D = \frac{c}{\pi}$$

D = ?"
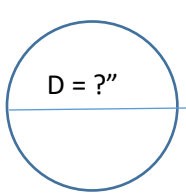
$$= \frac{16}{3.14}$$

= 5.09, which rounds to 5.1" to the nearest tenth.

4. We know that the radius = 5", and we are looking for area. The formula for area of a circle is:
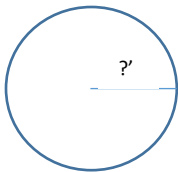
Area = πr^2

5"

= (3.14)(5)(5)   Fill in the known information; r=5; π=3.14

= 78.5 square inches.   Be sure to round to nearest tenth and use the unit of measure, square inches.

5. We are given the area, and we are going to find the radius.

|		
Area = πr^2	Write out the formula for area of a circle.
200.96 = (3.14)r^2	Fill the equation with known information.
64 = r^2	Divide both sides by 3.14
8' = r	Find square root of both sides. Unit of measure is feet.

?'

6.
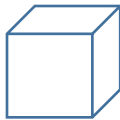
The volume of a cube is s^3. All sides of a cube are equal, so we multiply that measure by itself three times. (1)(1)(1) = 1. In this problem, the unit of measure is cubic inches, so the complete answer is 1 cu. in. The definition of a cubic inch is a shape that is 1" by 1" by 1".

7. The son's building blocks are exactly 1 cubic inch, and there are 500 blocks, so we need a box that will hold 500 cubic inches. The formula is length x width x height.

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10"   5"
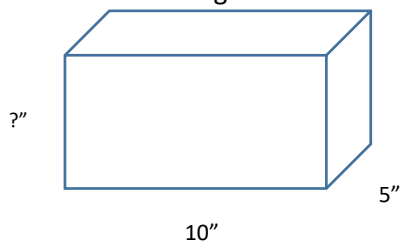
Volume of box = l x w x h
500 = 10 x 5 x h   Fill in what we know.
500 = 50h   Multiply 10 x 5 = 50.
10" = h   Divide both sides by 50.

8.

r = 1.25"

6"
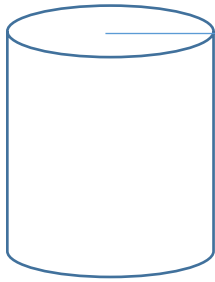
Start with a picture of the cylinder and what we know. First we need to find the volume of the cylinder. The formula is $\pi r^2 h$.

Volume = $\pi r^2 h$
         = (3.14)(1.25)(1.25)(6)
         = 29.44 cubic inches

The final piece of information we need is how many fluid ounces will fit in 29.44 cu in, given the proportion that each fluid ounce will occupy 1.8 cubic inches.

$\frac{fluid\ ounce}{cubic\ inch} = \frac{1}{1.8} = \frac{x}{29.44}$ Set up the proportion.

1.8x = 29.44   Cross-multiply.
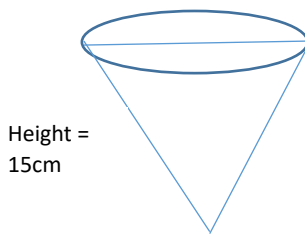x = 16.3   Divide each side by 1.8.
x = 16 fluid ounces   Round to the nearest fluid ounce.

9. We need to find the volume of the ice cream cone.

Diameter = 8cm, so radius = 4cm

Height = 15cm

Volume = $\frac{1}{3} \pi r^2 H$

= $\frac{1}{3}$ (3.14)(4)(4)(15)   Fill in knowns.

= 251.2 cubic cm.   Multiply.

10. A ball is a sphere, and the volume of a sphere is $\frac{4}{3} \pi r^3$.

We are given circumference as 10", so first we need to find radius.
Circumference = πD
10 = (3.14)D
3.18 = Diameter
1.59 = radius ($\frac{1}{2}$ of diameter)

Volume = $\frac{4}{3} \pi r^3$
= $\frac{4}{3}$ (3.14)(1.59)(1.59)(1.59)
= 16.8 cubic inches.

11. Volume of Pyramid = $\frac{1}{3}$ (length) (width) (height) = $\frac{1}{3}$ (9) (12) (6) = 216 cubic centimeters.