Exponents that are Integers

Exponents, also called "powers," show repeated multiplication of a "base" number. A base number of 4 and an exponent of 2 is written as 4^2, and we say "four to the second power." It means to multiply two bases together. So $4^2 = 4 \times 4 = 16$. To raise a base to the second power is also to "square" it, so 4^2 is also called "four squared."

To raise a base number to the third power is to "cube" it. 4^3 is "four to the third power," or "four cubed." The third power is when three bases are multiplied together: $4^3 = 4 \times 4 \times 4 = 64$.

If no exponent is shown, it is assumed to be 1. $4 = 4^1$. Any number raised to the first power is itself.

Any number raised to the zero power = 1. $4^0 = 1$.

A negative exponent means to raise the reciprocal of the base to the exponent. A reciprocal (also called multiplicative inverse) of a number is the value that can be multiplied by that number to get a product of one. It is found by inverting (or "flipping") the fraction form of a number. The reciprocal of 4 is $\frac{1}{4}$, because, written as a fraction, 4 is $\frac{4}{1}$. So $4^{-1} = \frac{1}{4^1} = \frac{1}{4}$.

$4^3 = 4 \times 4 \times 4 = 64$

$4^2 = 4 \times 4 = 16$

$4^1 = 4$

$4^0 = 1$

$4^{-1} = \frac{1}{4^1} = \frac{1}{4}$

$4^{-2} = \frac{1}{4^2} = \frac{1}{4 \, x \, 4} = \frac{1}{16}$

$4^{-3} = \frac{1}{4^3} = \frac{1}{4 \, x 4 \, x \, 4} = \frac{1}{64}$

Exponents that are Fractions

Exponents that are fractions are called "roots." The square root of 4 is commonly written as $\sqrt{4}$, but it is also $4^{\frac{1}{2}}$ or $\sqrt[2]{4^1}$. When the root symbol "√" is shown without a number outside, it is understood to be a square root.

The square root of four is the number that is multiplied by itself to get a result of 4. 2 x 2 = 4, AND -2 x -2 = 4. So $\sqrt{4} = 2$, AND $\sqrt{4} = -2$. That can be written $\sqrt{4} = \pm 2$, pronounced "plus or minus two," or "positive or negative two."

There are also "third" or "cube" roots $\sqrt[3]{\ }$, "fourth" roots $\sqrt[4]{\ }$, and so on. A cube root is the base such that three are multiplied together to get that product. For example, $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$, because 3 x 3 x 3 = 27.

Exponents can be any fraction. The standard format is $base^{\frac{n}{d}} = \sqrt[d]{base^n}$, where d is the denominator and n is the numerator of the fraction.

For example: $8^{\frac{2}{3}} = \sqrt[3]{8^2}$	"Eight to the two thirds power" is the cube root of eight squared.
$= \sqrt[3]{8 \, x \, 8}$	Write out 8^2 as 8 x 8.
$= \sqrt[3]{64}$	8 x 8 = 64
$= \sqrt[3]{4 \, x \, 4 \, x \, 4}$	Factor 64 as 4 x 4 x 4, which is 4^3
= 4	The cube root of four cubed is four.

Exponent Operations

Here are three basic rules for working with exponents:

$(x^a)(x^b) = x^{a+b}$	When multiplying the same base, add the exponents.
$\frac{x^a}{x^b} = x^{a-b}$	When dividing the same base, subtract the exponents.
$(x^a)^b = x^{ab}$	When raising an exponent to another exponent, multiply the exponents together.

Here are examples of each rule:

When multiplying the same base, add the exponents.

$(5^2)(5^3) = 5^{2+3} = 5^5$

(5 x 5)(5 x 5 x 5)	Write out 5^2 and 5^3
5 x 5 x 5 x 5 x 5	The product of five fives is 5^5
5^5	

When dividing the same base, subtract the exponents:

$\frac{4^7}{4^2} = 4^{7-2} = 4^5$

$\frac{4 \, x \, 4 \, x \, 4 \, x \, 4 \, x \, 4 \, x \, \cancel{4 \, x \, 4}}{\cancel{4 \, x \, 4}}$	Write out 4^7 and 4^2. Cancel out 4 x 4 from top and bottom.
4^5	The product of five fours is 4^5

When raising an exponent to another exponent, multiply the exponents together.

$(3^2)^4 = 3^8$

(3 x 3)(3 x 3)(3 x 3)(3 x 3) = 3^8 Write out 3^2 four times. The product of eight threes is 3^8

Multiply Polynomials

Monomials, like (2x), have one term. Binomials, like (x + 7), have two terms. Trinomials, like (x^2 + 6x – 7), have three terms. Polynomials have "many" terms, so they include all expressions with two or more terms. Review Section 16 on Factoring and FOIL if needed.

FOIL (first, outer, inner, last) is a checklist for multiplying two binomials to be sure that we multiply all terms together. To multiply polynomials, no matter how many terms there are, we have to multiply each term in the first group by each term in the second group.

For example, multiply:

$(2x)(4x^2 + 9x - 3)$

$(2x)(4x^2) + (2x)(9x) - (2x)(3)$ Distribute (2x) with each term.

$8x^3 + 18x^2 - 6x$ Multiply. Don't forget that $x = x^1$, so $(x)(x^2) = x^3$.

Multiply:

$(x + 2)(x^2 - 2x + 3)$ Multiply each term of first polynomial with each

$(x)(x^2) + (x)(-2x) + (x)(3) + (2)(x^2) + (2)(-2x) + (2)(3)$ term of the second polynomial.

$X^3 - 2x^2 + 3x + 2x^2 - 4x + 6$ Combine like-terms

$X^3 - x + 6$

Simplify Polynomials

Polynomials can be simplified by "canceling out" common factors. Any number divided by itself equals one, and dividing by one does not change an expression's value.

Simplify:

$\frac{x^2+x-2}{x-1}$ Use FOIL to factor into two binomials.

$\frac{(x+2)\cancel{(x-1)}}{\cancel{x-1}}$ One of the binomials is the same as the denominator. It cancels

X + 2 out because anything divided by itself = 1.

Simplify:

$\frac{-18x^4y^2}{3x^2y}$ (-18) ÷ 3 = -6; $x^4 \div x^2 = x^2$; $y^2 \div y = y$

$-6x^2y$

Scientific Notation

Scientific notation uses exponents and powers of 10 to make very large and very small numbers easier to compare. Any number can be formatted in scientific notation.

First, move the decimal point to the right or left to get a base number ≥ 1 and < 10. Then write the power of ten as "x 10^p" where p is the number of decimal places moved. If the decimal point moved to the right, p is negative; if the decimal moved to the left, p is positive. The larger the power of 10, the larger the number; the smaller the power of 10, the smaller the number.

For example: 1,432,759.32 = 1.43275932 x 10^6. The decimal point moved six places to the left.

For example: 0.00489 = 4.89 x 10^{-3}. The decimal point moved three places to the right.

Scientific notation can also make operations with large and small numbers easier. For example:

300 x .25 = 75. In scientific notation, that is:

$(3 \times 10^2)(2.5 \times 10^{-1}) = 7.5 \times 10^1$ 3 x 2.5 = 7.5; add the exponents to multiply the same base.

Logarithms

A logarithm is an exponent. The standard format for a logarithm is if $x = b^y$, then $y = \log_b x$. b is the base; y is the exponent, and x is the solution. The logarithm is the exponent to which the base must be raised to get the given number, x.

For example, $\log_3 27 = x$ Read "log base 3 of 27 equals x."

27 = 3x Rewrite using standard format $y = \log_b x$, means $x = b^y$

$3^3 = 3^x$ 27 = 3 x 3 x 3, which is 3^3.

3 = x 3 is the power to raise the base to get the value 27.

If no base is indicated, it is assumed to be 10, also called a "common logarithm."

For example, log x = 3 means $\log_{10} x = 3$

$10^3 = x$ the base, 10, raised to the exponent 3 = x

1000 = x 10^3 = 10 x 10 x 10 = 1000

Logarithms can also include decimals. They help compare and perform calculations with large and small numbers. This section is only an introduction to the use of logarithms.

Practice Problems

1. $4^3 + 5^2$ 2. $16^{\frac{1}{2}} - 3^0$ 3. $\sqrt{(5^2)(2^2)}$ 4. $4^{-2} + 2^{-4} + 2^3$

5. $25^{-\frac{1}{2}}$ 6. $6(6^3)$ 7. $\frac{5^3}{5^2}$ 8. $(4^2)^2$

9. $\left(\frac{36}{16}\right)^{-\frac{1}{2}}$ 10. $4y(y^4 - y^2 + 3)$ 11. $\frac{(2x^2-2)}{x+1}$ 12. $\frac{24x^3y^2}{6x^2y}$

13. $\frac{9m^2}{4} \div \frac{3m}{24}$

Put in scientific notation: 14. .000425 15. 5,844 16. 3,500,000 ÷ .07

Solve: 17. Log x = 4 18. $\text{Log}_2\ 16^{x+1} = n$

Answer Key for Practice Problems

1. 89	2. 3 OR -5	3. ±10	4. $8\frac{1}{8}$
5. $\frac{1}{5}$ OR $\frac{-1}{5}$	6. 1, 296	7. 5	8. 256
9. $\pm\frac{2}{3}$	10. $4y^5 - 4y^3 + 12y$	11. 2(x – 1)	12. 4xy
13. 18m	14. 4.25×10^{-4}	15. 5.844×10^3	16. $.5 \times 10^8$
17. 10,000	18. n = 4x + 4		

Practice Problems Solved with Explanation

1. $4^3 + 5^2$
   64 + 25
   89

4^3 means the product of three bases, 4, which is 4 x 4 x 4 = 64.
5^2 means the product of two bases, 5, which is 5 x 5 = 25.

2. $16^{\frac{1}{2}} - 3^0$
   $\sqrt{16}$ - 1
   4 – 1 OR -4 -1
   3 OR -5

Fractional exponents mean $base^{\frac{n}{d}}$ = $\sqrt[d]{base^n}$, so the exponent $\frac{1}{2}$ means $\sqrt[2]{16^1}$, or the square root of 16 to the first power. The square root of 16 is 4 or -4, so there are two possible answers. 3^0 = 1 because any number to the 0 power equals 1.

3. $\sqrt{(5^2)(2^2)}$
   $\sqrt{(25)(4)}$
   $\sqrt{100}$
   ± 10

First raise 5 to the 2nd power; 5 x 5 = 25; 2^2 = 2 x 2 = 4. Multiplying 25 x 4 = 100. The square root of 100 is the number that is multiplied by itself to get 100. Both 10 and -10 are the square roots of 100.
We could have also found that the square root of 5^2 = ±5, and the square root of 2^2 = ±2. Multiplying (5 x 2) or (-5 x 2) would also get ±10.

4. $4^{-2} + 2^{-4} + 2^{3}$

$\frac{1}{4^2} + \frac{1}{2^4} + 8$

$\frac{1}{16} + \frac{1}{16} + 8$

$8\frac{2}{16}$

$8\,\frac{1}{8}$

A negative exponent means to take the reciprocal of the base raised to to the exponent. $4^2 = 4 \times 4 = 16$. $2^4 = 2 \times 2 \times 2 \times 2 = 16$.

Add the results and simplify the fraction.

5. $25^{-\frac{1}{2}}$

$\frac{1}{\sqrt{25}}$

$\frac{1}{5}$ OR $\frac{-1}{5}$

A negative exponent means to take the reciprocal raised to the exponent. The exponent is a fraction, so the numerator is the power and the denominator is the root, in this case $\sqrt[2]{25^1}$, or $\sqrt{25}$.

Both 5 and -5 are square roots of 25, so the answer is $\frac{1}{5}$ OR $\frac{-1}{5}$.

6. $6(6^3)$

$6^1(6^3)$

6^{1+3}

6^4

6 x 6 x 6 x 6

1,296

If no exponent is written, it is assumed to be 1, so $6 = 6^1$.
Following the rules for exponent operations, add the exponents when multiplying the same base.

7. $\frac{5^3}{5^2}$

5^{3-2}

$5^1 = 5$

When dividing the same base, subtract the exponents.

Any number raised to the power of 1 is itself.

8. $(4^2)^2$

4^4

4 x 4 x 4 x 4 = 256

To raise an exponent to an exponent, multiply the exponents together.

9. $\left(\frac{36}{16}\right)^{-\frac{1}{2}}$

$\sqrt{\frac{16}{36}}$

$\pm\frac{4}{6} = \pm\frac{2}{3}$

Because the exponent is negative, we take the reciprocal, or "flip" the fraction. The exponent ½ indicates the square root. $\sqrt{16} = \pm 4$, and $\sqrt{36} = \pm 6$. The fraction simplifies to $\pm\frac{2}{3}$.

10. $4y(y^4 – y^2 + 3)$

$(4y)(y^4) – (4y)(y^2) + (4y)(3)$

$4y^5 – 4y^3 + 12y$

Each term inside the parentheses must be multiplied by 4y. Don't forget that the second term is negative! Add exponents together, so $4y^1(y^4) = y^5$.

11. $\frac{(2x^2-2)}{x+1}$ — Start by factoring 2 out of both terms in the numerator. Then use FOIL to

$\frac{2(x^2-1)}{x+1}$ — factor (x^2 – 1), which is the difference between squares. The factor (x + 1)

$\frac{2(\cancel{x+1})(x-1)}{\cancel{x+1}}$ — is in both the numerator and denominator, so it "cancels" out.

2(x - 1)

12. $\frac{24x^3y^2}{6x^2y}$ — Separate the fraction into numerical factor, x-factor, and y-factor, and

$\left(\frac{24}{6}\right)\left(\frac{x^3}{x^2}\right)\left(\frac{y^2}{y}\right)$ — it's easier to see how to simplify. Remember to subtract exponents when

4xy — dividing the same base.

13. $\frac{9m^2}{4} \div \frac{3m}{24}$ — To divide fractions, multiply by the reciprocal of the second number.

$\left(\frac{9m^2}{4}\right)\left(\frac{24}{3m}\right)$ — There are several common factors that can be removed to simplify.

$\frac{(3)\cancel{(3)m}m(6)\cancel{(4)}}{\cancel{(4)}\cancel{(3)m}}$ — Remember, that any number divided by itself, like $\frac{m}{m}$, equals 1.

18m

14. .000425 — Move the decimal point to the right until there is just one number before it.

4.25×10^{-4} — Since it moved four places to the right, the exponent is -4.

15. 5,844 — Move the decimal point three places to the left to get 5.844. The exponent is 3.

5.844×10^3

16. 3,500,000 ÷ .07 — Convert both numbers to scientific notation.

$(3.5\text{x}10^6) \div (7\text{x}10^{-2})$ — 3.5 ÷ 7 = .5

$.5 \times 10^8$ — Subtract the exponents when dividing by the same base. (6) – (-2) = 8

17. Log x = 4 — When the logarithm does not specify a base, it is assumed to be 10.

$X = 10^4$ — A logarithm is an exponent; since the log x is 4, 4 is the exponent.

X = 10,000 — The base, 10, to the 4^{th} power = 10,000.

18. $\log_2 16^{x+1} = n$ — $\log_2$ means the base is 2. The logarithm is the exponent; n is the exponent.

$2^n = 16^{x+1}$ — So, rearranging things, we get base 2 to the nth power, which is written 2^n.

$2^n = (2^4)^{x+1}$ — Rewrite 16 with a base of 2, which is 2^4.

$2^n = 2^{4x+4}$ — Multiply the exponents together.

n = 4x + 4 — The nth power of 2 is the same as the (4x + 4)th power of 2, so n = 4x + 4.