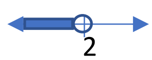
Graphing Inequalities

Inequalities with one variable can be plotted on a number line, shown in Section 8. Inequalities can also have two variables and be graphed almost exactly like linear equations.

When inequalities are plotted on a number line, the endpoint is a solid circle to indicate an "or equal to" option. The number line below on the left shows the inequality X ≥ 2. On the right below is the example x < 2, where an open circle on the endpoint indicates that 2 is not included.

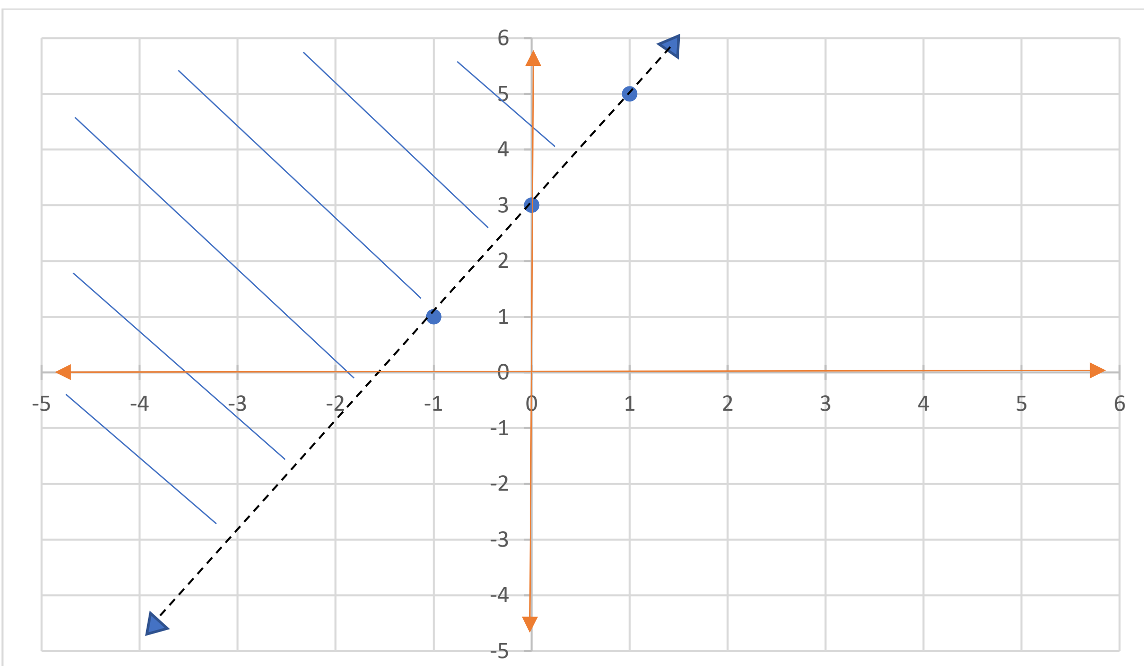
x ≥ 2



x < 2

When an inequality is graphed using x- and y-coordinates, a solid line indicates the "equal to" option, meaning that all points on the line are included in the solution. A dotted line means the points on the line are not included in the solution. And like the number line is shaded to show what range of numbers is included, the graph is shaded to indicate where all the coordinates satisfy the inequality.

Below is the graph of the inequality y > 2x + 3. That equation is in the slope-intercept format. The y-intercept is 3, and the slope is 2. The line is dotted because there is a > (greater than) sign, not ≥ (greater than or equal to). The area to the left of the line is shaded. For the equation y = 2x + 3, only the points on the line would be part of the solution set. But in an inequality, all the points in the shaded portion of the graph are part of the solution.

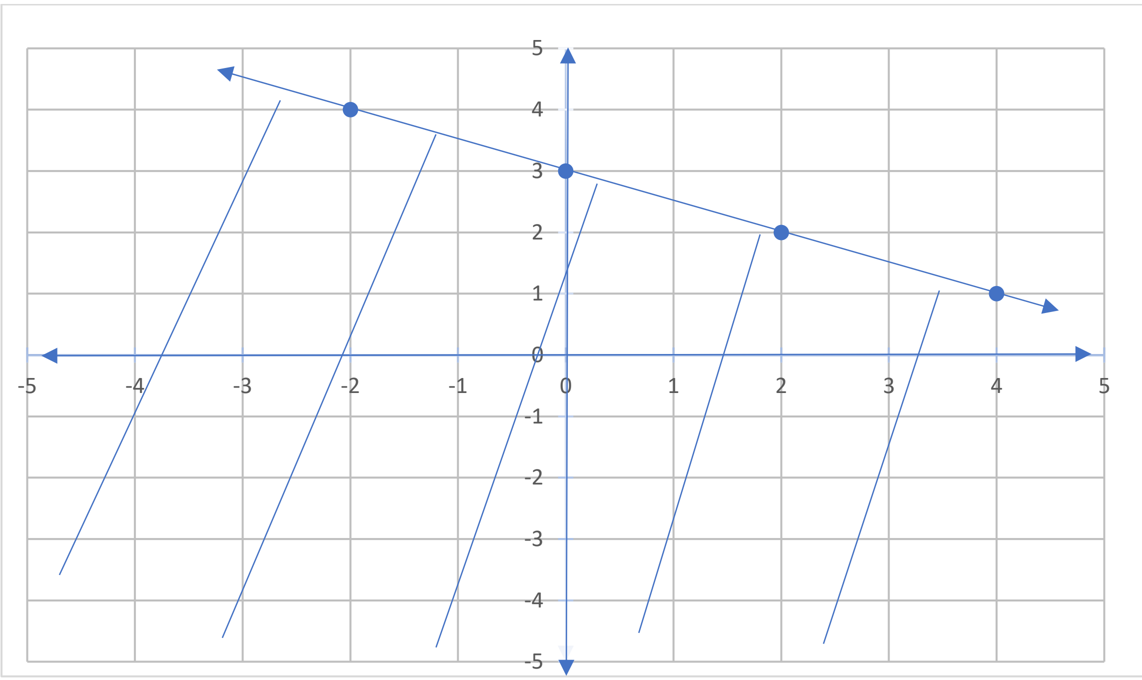


For example, the point (-3,3) is not on the line, but it is in the shaded portion. When we test those coordinates in the inequality, the result is true.

y > 2x + 3
3 > 2(-3) + 3   Substitute (-3,3) to test the coordinates.
3 > -3   It is true that 3 is greater than -3, so point (-3,3) is part of the solution.

Testing coordinate values determines which side of the line should be shaded. When the point's coordinates make the inequality true, that side of the graph is shaded. We can also tell which side of the graph to shade when the inequality is in standard slope-intercept format (y = mx+b). Since the inequality in this case (y > 2x + 3) includes "is greater than," the side of the line that includes y-values greater than the intercept should be shaded.

Here is the graph of $y \leq \frac{-1}{2}x + 3$.



In this graph, the y-intercept is 3 and the slope is $\frac{-1}{2}$. The line is solid because the symbol ≤ means "less than or equal to," so the points on the line are part of the solution. When we test the point (0,0) it makes the inequality true, so that side of the graph is shaded.

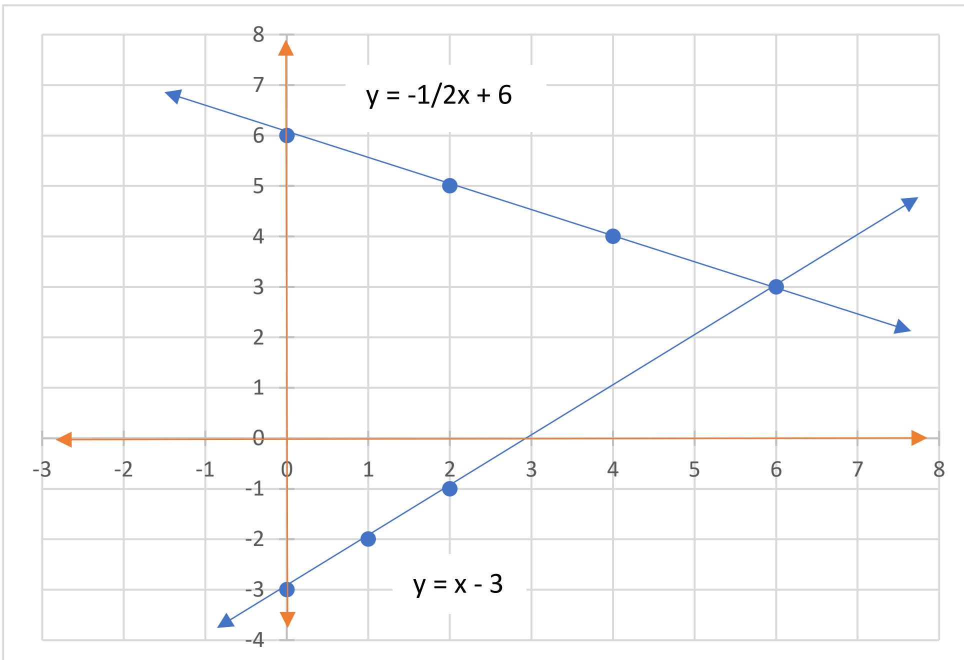
$y \leq \frac{-1}{2}x + 3$
$0 \leq 0 + 3$
$0 \leq 3$

Systems of Equations Solved with Graphing

A system of equations is a set of two (or more) equations with the same variables. If the two equations do not intersect, there is no common solution. Parallel lines, for example, never intersect. Other pairs of linear equations will intersect at one point. There are three ways to find the point where the lines intersect, which is the solution of the system.

First, we can graph both lines and determine from the graph the coordinates where the lines intersect.

Here is the graph of the system of equations: $y = x - 3$ and $y = \frac{-1}{2}x + 6$:



The graph shows the two lines intersecting at point (6,3). To verify that this point is the solution to both equations, test the coordinates.

$y = x - 3$	$y = \frac{-1}{2}x + 6$	When the coordinates of (6,3) are substituted
$3 = 6 - 3$	$3 = \frac{-1}{2}(6) + 6$	into both equations, the results are "true."
$3 = 3$	$3 = -3 + 6$	So (6,3) is the solution of this system of
	$3 = 3$	equations.

This method may be more difficult to solve when the solution has coordinates that contain fractions or decimals or when the solution contains large numbers. It can also be time-consuming to graph both equations.

Systems of Equations Solved with Substitution

Another method to solve a system of equations is called "substitution." In this method, first one equation is solved in terms of the other (like y = x-3), then that equivalent expression is substituted into the other equation.

In the system of equations y = x - 3 and y = $\frac{-1}{2}$x + 6, the first equation is already solved for y in terms of x. We can substitute (x – 3) for y in the second equation:

$y = \frac{-1}{2}x + 6$	Original equation
$x - 3 = \frac{-1}{2}x + 6$	Since y = x – 3, substitute x – 3 for y.
$\frac{3}{2}x = 9$	Add $\frac{1}{2}$x to both sides and add 3 to both sides.
$3x = 18$	Multiply both sides by 2.
$x = 6$	Divide both sides by 3.

After solving for the first variable, in this case finding that x = 6, we substitute that value into one of the original equations to solve for y.

$y = x - 3$	Original equation
$y = 6 - 3$	Since x = 6, substitute 6 for x.
$y = 3$	Solve for y.

The solution to this system of equations is the point with coordinates (6,3).

System of Equations Solved with Elimination

A third method to solve a system of equations is to eliminate one variable by adding (or subtracting) one equation with the other. One of the two variables must have the same (or opposite) coefficient in both equations. Add or subtract to "cancel" or "eliminate" that variable.

In the system of equations y = x -3 and y = $\frac{-1}{2}$x + 6, the y variable has a coefficient of 1 in both equations.

$y = x - 3$	Line equations up with y-term, x-term, and numeric-term in the same order.
$\underline{-(y = \frac{-1}{2}x + 6)}$	
$0 = \frac{3}{2}x - 9$	Subtract each term: (y) – (y) = 0; (x) - ($\frac{-1}{2}$x) = $\frac{3}{2}$x; and (-3) – (6) = -9
$9 = \frac{3}{2}x$	Add 9 to both sides.
$6 = x$	Divide both sides by $\frac{3}{2}$.

Just like in the substitution example, after solving for the first variable, use that value to find that y = 3.

Both substitution and elimination are possible for all systems of equations, but usually one or the other method will require less manipulation of the original equations.

Example: Solve system of equations: $y = 3x - 12$; and $-6y = -3x + 12$

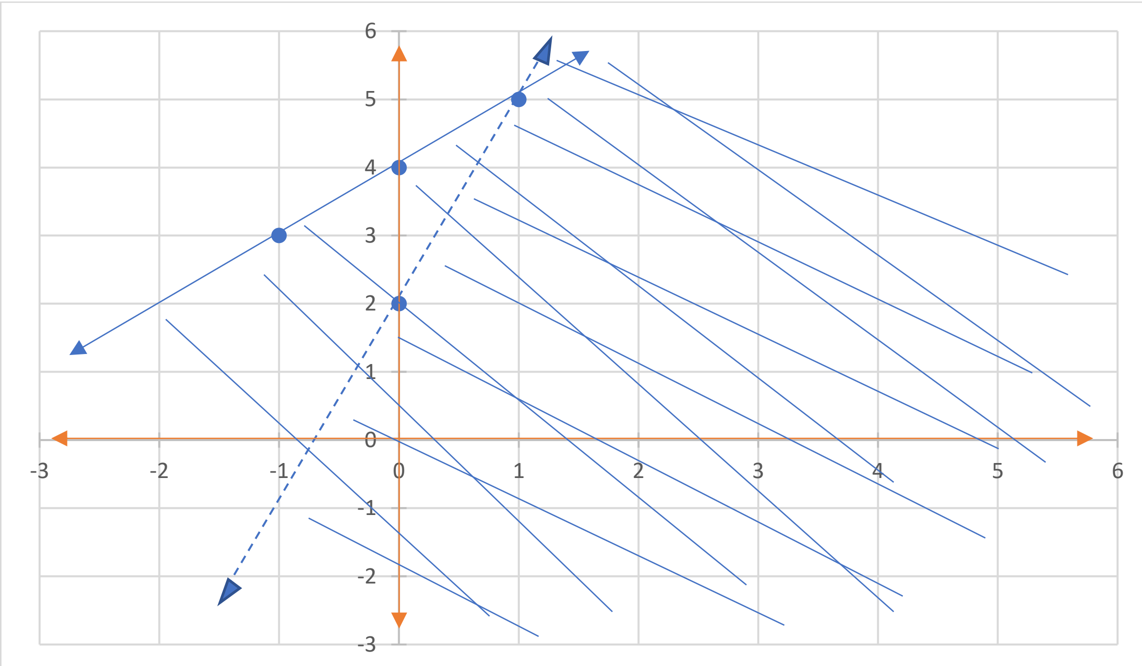
Substitution: Let $y = 3x - 12$	Elimination: add equations together
$-6(3x - 12) = -3x + 12$	$(\ y = 3x - 12)$
$-18x + 72 = -3x + 12$	$\underline{+ (-6y = -3x + 12)}$
$72 = 15x + 12$	$-5y = 0$
$60 = 15x$	$y = 0$
$4 = x$	
$y = 3x - 12$	$y = 3x - 12$
$y = 3(4) - 12$	$0 = 3x - 12$
$y = 12 - 12$	$12 = 3x$
$y = 0$	$4 = x$
Solution: (4, 0)	Solution: (4, 0)

Example: Solve system of equations: $y - 19 = 3x$; and $3y = -6x -3$

Substitution:	Elimination:
Solve for y: $y - 19 = 3x$ and get	To get same coefficient:
$y = 3x + 19$	Multiply $2(y - 19) = 2(3x)$
Substitute into $3y = -6x - 3$	$2y - 38 = 6x$
$3(3x + 19) = -6x - 3$	Rearrange to line variables up
$9x + 57 = -6x - 3$	$(2y = 6x + 38)$
$15x = -60$	$\underline{+(3y = -6x - 3)}$ add equations together
$x = -4$	$5y = 35$
$3y = -6(-4) - 3$	$y = 7$
$3y = 21$	$7 - 19 = 3x$
$y = 7$	$-12 = 3x$
Solution (-4, 7)	$-4 = x$
	Solution (-4,7)

Solving a System of Inequalities

We can also solve a system of inequalities, but the solution will contain many points, not just one point. Start by graphing the two inequalities: $y \le x + 4$ and $y < 3x + 2$



First look at the line $y \le x + 4$. It is a solid line because the "equal to" option is included. The graph is shaded to the right of that line. Then look at the line for $y < 3x + 2$ It is a dotted line because the points can not be equal to that value. The graph is also shaded to the right of that line.

Even though the lines intersect at (1,5) those coordinates are NOT part of the solution. This can be tested by substituting them in the $y < 3x + 2$ inequality:

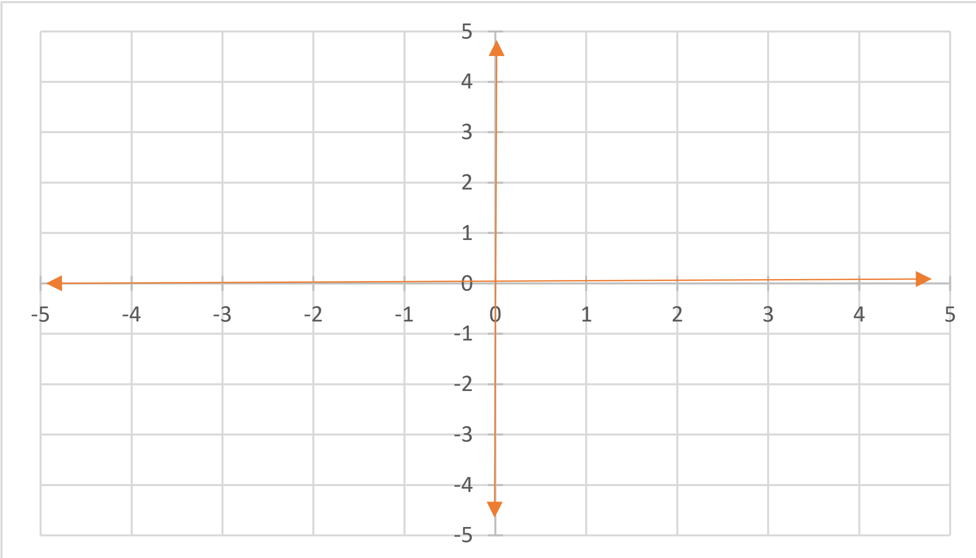
$y < 3x + 2$
$5 < 3(1) + 2$
$5 < 5$ NOT true

The solution is all the points in the area where the shading overlaps. To confirm that a point in that area is part of the solution set, substitute the coordinates into both inequalities and confirm that the results are true. For example, point (0,0) is in the shaded part of both graphs.

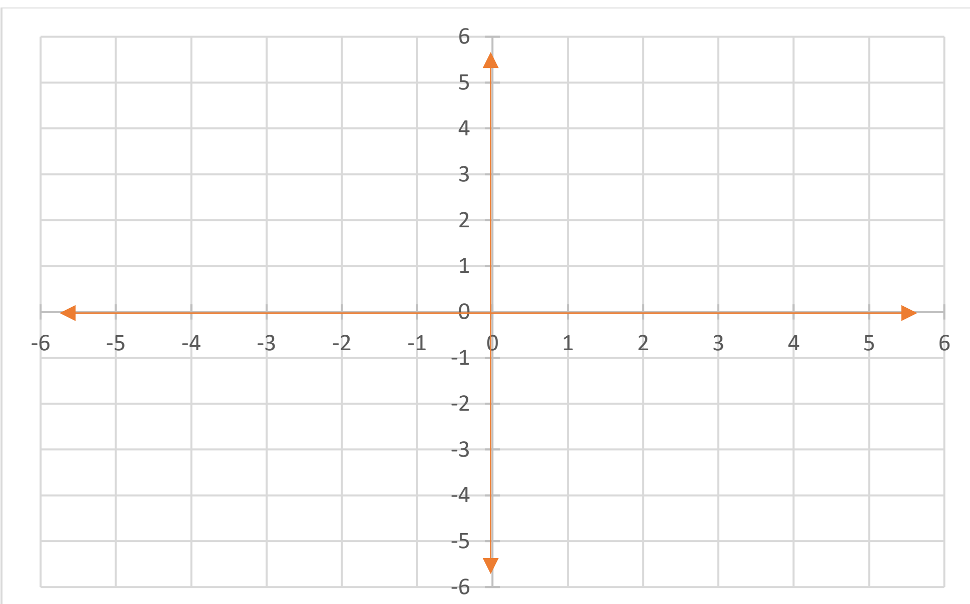
$y \le x + 4$	$y < 3x + 2$
$0 \le 0 + 4$	$0 < 3(0) + 2$
$0 \le 4$ TRUE	$0 < 2$ TRUE

Practice Problems

1. Graph the inequality y < 2x – 1. Confirm whether the point (1, 1) is part of the solution.



2. Graph the system of equations: y = -2x -4 and y = x + 5. Solve with substitution and elimination.



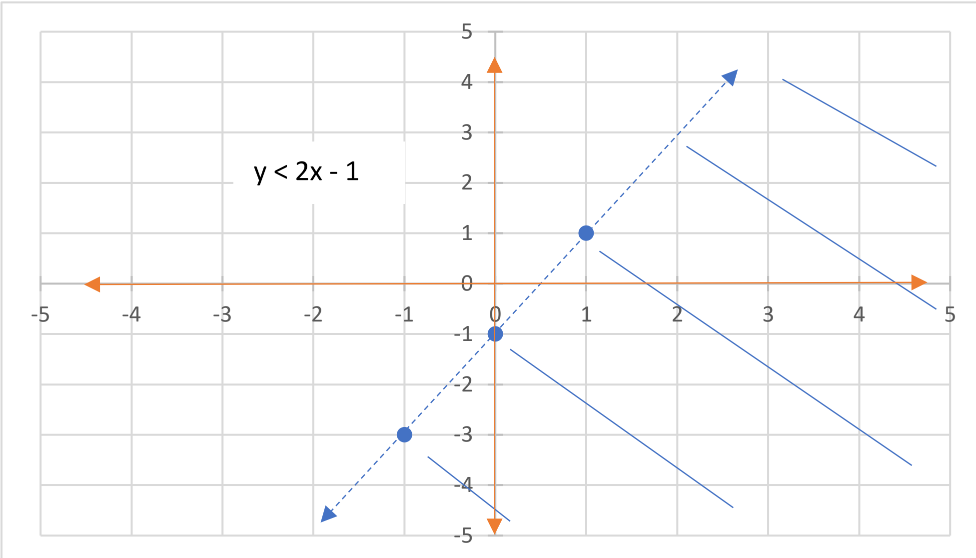
3. Solve with elimination: 3y = 5x + 3; and 4y = 5x + 8

4. Solve with substitution: 2y = -4x + 6 and 5y + 6 = 4x

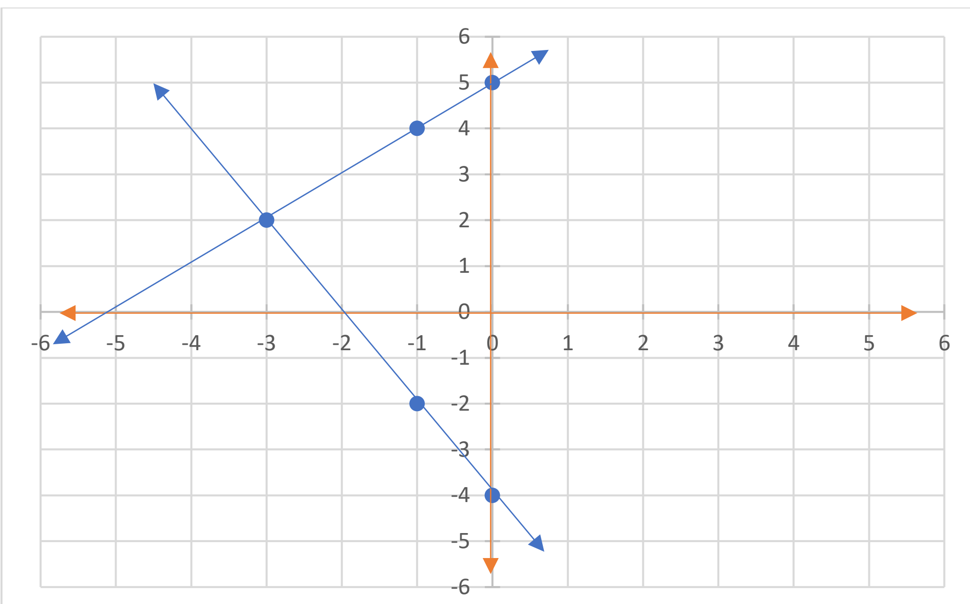
5. The sum of two numbers is 36. If the larger number is 3 times the smaller number, what are the two numbers? Solve with a system of equations.

Answer Key for Practice Problems

1. (1, 1) is not part of the solution.



2. (-3,2)



3. $(2\frac{2}{5}, 5)$ 4. $(\frac{3}{2}, 0)$ 5. 9, 27

Practice Problems Solved with Explanation

1. The y-intercept is -1, and the slope is 2. The line is dashed because the inequality is "less than," and it does not include the "equal to" option. The graph is shaded to the right because that includes the y-values that are less than the line. We can also test point (0, 0) to show it is not on the line.

$y < 2x - 1$	
$0 < 2(0) - 1$	substitute values (0, 0) into inequality
$0 < -1$	0 is not less than -1, so that point is not in the solution set; shade the opposite side

Point (1, 1) is on the line, but when we test its coordinates, the results are not true.

$y < 2x - 1$	
$1 < 2(1) - 1$	substitute values (1, 1) into inequality
$1 < 1$	1 is not less than 1, so it is not part of the solution.

2. $y = -2x - 4$, and $y = x + 5$. The graph shows that they intersect at the point (-3, 2)

Solving with substitution:

$-2x - 4 = x + 5$	Both equations show y in terms of x, so the x-expressions are equal to each other.
$-9 = 3x$	Subtract 5 from both sides, and add 2x to both sides.
$-3 = x$	Divide both sides by 3.
$y = x + 5$	
$y = (-3) + 5$	Substitute -3 for x in one of the original equations
$y = 2$	X = -3 and y = 2, so the coordinates of the solution are (-3, 2).

Solving with elimination:

$y = -2x - 4$	Original equation
$\underline{2y = 2x + 10}$	This is the original equation (y = 2 + 5) multiplied by 2. This gives opposite coefficients
$3y = 6$	for the x-term. Add the equations together, which eliminates the x-term.
$y = 2$	Divide both sides by 3.
$y = x + 5$	
$2 = x + 5$	Substitute 2 for y in one of the original equations.
$-3 = x$	x = -3 and y = 2, so the coordinates of the solution are (-3, 2).

3. $3y = 5x + 3$	Both equations have x-coefficient = 5, so if one equation is subtracted from the
$\underline{-\ 4y = 5x + 8}$	other, the x-term will be eliminated. Be careful to subtract each term.
$-y = -5$	Divide both sides by -1
$y = 5$	

$3y = 5x + 3$	
$3(5) = 5x + 3$	Substitute -1 for y in one of the original equations.
$15 = 5x + 3$	
$12 = 5x$	Subtract 3 from both sides
$2\frac{2}{5} = x$	Divide both sides by 5 and simplify the equation.

4. $2y = -4x + 6$	Divide all terms of this equation, on both sides, by 2 to solve for y.
$y = -2x + 3$	
$5y + 6 = 4x$	
$5(-2x + 3) + 6 = 4x$	Substitute $(-2x + 3)$ for y in the other equation.
$-10x + 15 + 6 = 4x$	Add 10x to both sides
$21 = 14x$	Divide both sides by 21 and simplify the fraction.
$\frac{3}{2} = x$	
$y = -2x + 3$	
$y = -2(\frac{3}{2}) + 3$	Substitute $\frac{3}{2}$ for x in original equation
$y = -3 + 3$	
$y = 0$	

5. Let x = one number and let y = the other number.

$X + y = 36$	The sum of the two numbers equals 36.
$x = 3y$	One of the numbers is three times the other number.
$3y + y = 36$	Substitute 3y for x
$4y = 36$	$3y + y = 4y$
$y = 9$	Divide both sides by 4
$X + y = 36$	
$x + 9 = 36$	Substitute 9 for y in one of the original equations.
$x = 27$	Subtract 9 from both sides.