SECTION 2 – FRACTIONS

A fraction is made up of $\frac{numerator}{denominator}$. For example, $\frac{1}{4}$ means that something has been divided into four parts, and we have one part. $\frac{1}{4}$ is also the same as 1 divided by 4.

Any integer can be in fraction format when the integer is the numerator and the denominator is 1. For example, 3 can be written as $\frac{3}{1}$.

A fraction can not have 0 as the denominator. $\frac{1}{0}$ is "undefined." That would mean "one divided by zero," and dividing by 0 is not possible.

Multiply Fractions

To multiply fractions, the numerator is the product of the numerators, the denominator is the product of the denominators. In other words, multiply straight across:

$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12}$

Simplify Fractions (including "Canceling")

We usually want to simplify fractions, which means that our final result should not have any common factors in both the top (numerator) and bottom (denominator). In the example $\frac{6}{12}$, we know that 6 and 12 both share a factor of 6. So we can divide both the numerator and denominator by 6 to get the simplified form:

$\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}$

Some people use the phrase "cancel out" when they talk about simplifying fractions. That means to factor the top and the bottom; find common factors; and cross, or "cancel" them out.

$\frac{6}{12} = \frac{\cancel{6} \times 1}{\cancel{6} \times 2}$ We "cancel out" the $\frac{6}{6}$ part (because any number divided by itself = 1) and get $\frac{1}{2}$.

We can do some "canceling" before we multiply if we factor first. For example:

$\frac{3}{4} \times \frac{1}{6} = \frac{\cancel{3}}{4} \times \frac{1}{\cancel{3} \times 2} = \frac{1}{8}$

We factored the 6 as 3 x 2. Then any factor that appears in either numerator and either denominator can be "cancelled out."

Divide Fractions

Dividing fractions is the same as "multiplying by the reciprocal." A number's reciprocal is the number that results in a value of one when multiplied together. To find a fraction's reciprocal, just "flip" the numerator and denominator. So, to divide fractions, the second fraction is flipped over, and then multiply as usual. For example:

$\frac{5}{8} \div \frac{2}{3}$

$\frac{5}{8} \times \frac{3}{2} = \frac{15}{16}$

Add and Subtract Fractions

Fractions must have the same, or "common" denominator before addition and subtraction can occur. If the denominators are already the same, we just add the numerators and keep the denominator the same: $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

When the denominators are not already the same, we find a number that is divisible by both denominators. Convert each fraction to have that denominator by multiplying by one in the form of a fraction, where both numerator and denominator is that other factor.

For example: $\frac{1}{2} + \frac{1}{3}$ do not have the same denominator. We can convert each fraction to have the common denominator of 6 as follows:

$\frac{1}{2} \times \frac{3}{3} + \frac{1}{3} \times \frac{2}{2}$ Both starting denominators are factors of 6. We multiply top and

$\frac{3}{6} + \frac{2}{6}$ bottom by the other factor to get the common denominator 6.

$\frac{5}{6}$

Subtraction works the same:

$\frac{3}{4} - \frac{1}{2}$

$\frac{3}{4} - \frac{1}{2} \times \frac{2}{2}$ In this example, 4 is a common denominator, so only the second

$\frac{3}{4} - \frac{2}{4}$ fraction needed to change.

$\frac{1}{4}$

The "least common denominator" means the smallest value that can be used to make the denominators the same. Our answer would eventually be the same if we had used 8 (or 16, or 28, etc.) for a common denominator instead of 4.

$\frac{3}{4} - \frac{1}{2}$

$\frac{3}{4} \times \frac{2}{2} - \frac{1}{2} \times \frac{4}{4}$

$\frac{6}{8} - \frac{4}{8} = \frac{2}{8}$ which we simplify to $\frac{2 \div 2}{8 \div 2} = \frac{1}{4}$.

Same answer, but we had to simplify because we didn't start with the least common denominator.

Mixed Fractions and Proper Fractions

A "mixed" fraction has a whole number and a fraction. For example, read $3\frac{1}{5}$ as "three AND one fifth." So, we have three whole things, and one-fifth of a thing. Sometimes to add, subtract, multiply or divide mixed fractions they need to be converted into non-mixed format. A whole thing is all of its pieces. For fifths, each whole object is $\frac{5}{5}$.

So, $3\frac{1}{5}$ is the same as $\frac{5}{5} + \frac{5}{5} + \frac{5}{5} + \frac{1}{5}$ which equals $\frac{16}{5}$.

The same result is found by multiplying the whole number by the denominator then adding the numerator of the fraction. In this example 3 x 5 = 15; 15 + 1 = 16. So $3\frac{1}{5}$ converts to $\frac{16}{5}$.

The reverse also works to convert an "improper fraction" (where the numerator is bigger than the denominator) to its simplified version. Divide the numerator by the denominator. The "proper" mixed fraction is the whole number result with the remainder placed over the denominator.

For example: $\frac{14}{3} = 4\frac{2}{3}$. (14 ÷ 3 = 4, with a remainder of 2)

What's Bigger?

Many people think $\frac{1}{3}$ is smaller than $\frac{1}{4}$. That's because they know 3 is smaller than 4, and they jump to the wrong conclusion. The best way to compare fractions is to give them a common denominator.

Use 12 as the common denominator in this example: $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$ and $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

Now we can tell that 4 things is more than 3 of the same things. Therefore, $\frac{1}{3}$ is more than $\frac{1}{4}$.

Practice Problems

Simplify the following fractions:

1. $\frac{4}{12}$ 2. $\frac{5}{1}$ 3. $\frac{10}{15}$ 4. $\frac{20}{4}$ 5. $\frac{17}{3}$

Multiply the following, and simplify your answer if possible:

6. $\frac{1}{2} \times \frac{3}{5}$ 7. $\frac{3}{4} \times \frac{5}{12}$ 8. $1\frac{1}{2} \times 2$ 9. $3\frac{1}{8} \times 4\frac{2}{5}$

Divide the following, and simplify your answer if possible:

10. $\frac{2}{3} \div \frac{1}{2}$ 11. $5 \div \frac{3}{4}$ 12. $2\frac{1}{3} \div 3$ 13. $\frac{5}{12} \div \frac{1}{3}$

Which is bigger?

14. $\frac{2}{3}$ or $\frac{3}{4}$ 15. $\frac{3}{7}$ or $\frac{4}{9}$

Add the following, and simplify your answer if possible:

16. $\frac{1}{7} + \frac{4}{7}$ 17. $\frac{1}{6} + \frac{2}{3}$ 18. $\frac{4}{5} + \frac{5}{7}$ 19. $3\frac{1}{3} + 2\frac{1}{2}$

Subtract the following, and simplify your answer if possible:

20. $\frac{3}{4} - \frac{1}{4}$ 21. $\frac{5}{12} - \frac{1}{3}$ 22. $2 - \frac{5}{7}$ 23. $2\frac{5}{8} - 1\frac{2}{3}$

24. $\frac{6}{7} - \frac{1}{3}$

Answer Key for Practice Problems

1. $\frac{1}{3}$ 2. 5 3. $\frac{2}{3}$ 4. 5 5. $5\frac{2}{3}$ 6. $\frac{3}{10}$ 7. $\frac{5}{16}$ 8. 3 9. $13\frac{3}{4}$ 10. $1\frac{1}{3}$

11. $6\frac{2}{3}$ 12. $\frac{7}{9}$ 13. $1\frac{1}{4}$ 14. $\frac{3}{4}$ 15. $\frac{4}{9}$ 16. $\frac{5}{7}$ 17. $\frac{5}{6}$ 18. $1\frac{18}{35}$ 19. $5\frac{5}{6}$ 20. $\frac{1}{2}$

21. $\frac{1}{12}$ 22. $1\frac{2}{7}$ 23. $\frac{23}{24}$ 24. $\frac{11}{21}$

Practice Problems Solved with Explanation

1. $\frac{4}{12} = \frac{4 \div 4}{12 \div 4} = \frac{1}{3}$ 4 and 12 are both divisible by 4.

   We could also solve this problem with factoring and canceling: $\frac{4}{12} = \frac{\cancel{4} \, x \, 1}{\cancel{4} \, x \, 3} = \frac{1}{3}$

2. $\frac{5}{1} = 5$ When the denominator is 1, the expression can be simplified to just the numerator.

3. $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$ 10 and 15 are both divisible by 5.

4. $\frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1} = 5$ 20 and 4 are both divisible by 4

5. $\frac{17}{3} = 5\frac{2}{3}$ 17 ÷ 3 is 5 with a remainder of 2. 5 is the whole number. 2 is the remainder, which stays over the original denominator, 3.

6. $\frac{1}{2}$ x $\frac{3}{5} = \frac{3}{10}$ Multiply straight across, numerator times numerator; denominator times denominator.

7. $\frac{3}{4}$ x $\frac{5}{12}$ Factor the 12 as 3 x 4 first, then the threes can be canceled. Multiply straight across.

   $\frac{\cancel{3}}{4}$ x $\frac{5}{\cancel{3} \, x \, 4} = \frac{5}{16}$

8. $1\frac{1}{2}$ x 2 Convert the mixed number by multiplying the whole number 1 by the denominator 2, then add numerator 1. Result is $\frac{3}{2}$. The 2s can be canceled before multiplying straight across, then simplified to 3.

   $\frac{3}{\cancel{2}}$ x $\frac{\cancel{2}}{1} = \frac{3}{1} = 3$

9. $3\frac{1}{8} \times 4\frac{2}{5}$ Convert the mixed numbers first. Then factor before multiplying.

$\frac{25}{8} \times \frac{22}{5}$ There are matching 5s and 2s in top and bottom that can cancel.

$\frac{\cancel{5} \times 5}{\cancel{2} \times 4} \times \frac{\cancel{2} \times 11}{\cancel{5}}$ After multiplying, convert the improper fraction to a mixed number.

$\frac{55}{4} = 13\frac{3}{4}$ 55 ÷ 4 is 13 with a remainder of 3.

10. $\frac{2}{3} \div \frac{1}{2}$ Flip the second fraction over (to its reciprocal) and multiply

$\frac{2}{3} \times \frac{2}{1} = \frac{4}{3}$ straight across. Then change the improper fraction to a mixed number.

$1\frac{1}{3}$ 4 ÷ 3 = 1 with a remainder of 1.

11. $5 \div \frac{3}{4}$ Change the whole number to a fraction with denominator = 1. Flip

$\frac{5}{1} \times \frac{4}{3} = \frac{20}{3}$ the second fraction over and multiply straight across. Change the

$6\frac{2}{3}$ improper fraction to a mixed number. 20 ÷ 3 = 6 with remainder 2.

12. $2\frac{1}{3} \div 3$ Convert the mixed number. Remember that 3 is $\frac{3}{1}$ when written as a

$\frac{7}{3} \times \frac{1}{3} = \frac{7}{9}$ fraction, so its reciprocal is $\frac{1}{3}$. Multiply straight across.

13. $\frac{5}{12} \div \frac{1}{3}$ Factoring the 12 as 3 x 4 lets us cancel the 3s when the second

$\frac{5}{\cancel{3} \times 4} \times \frac{\cancel{3}}{1} = \frac{5}{4} = 1\frac{1}{4}$ fraction is flipped over. Multiply across, then convert to mixed number.

14. $\frac{2}{3}$ or $\frac{3}{4}$ Use 12 as the common denominator. Multiply each fraction by 1, in

$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$ the form of a fraction which has a top and bottom equal to the factor

$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$ needed for the result of 12 in the denominator. 9 is larger than 8.

15. $\frac{3}{7}$ or $\frac{4}{9}$

$\frac{3}{7} \times \frac{9}{9} = \frac{27}{63}$

$\frac{4}{9} \times \frac{7}{7} = \frac{28}{63}$

Use 63 as the common denominator. Multiply each fraction by 1, in the form of a fraction with top and bottom equal to the factor needed for the result of 63. 28 is larger than 27, so $\frac{4}{9}$ is larger.

16. $\frac{1}{7} + \frac{4}{7} = \frac{5}{7}$

The denominators are already the same, so add the numerators and keep the denominator.

17. $\frac{1}{6} + \frac{2}{3}$

$\frac{1}{6} + \frac{2}{3} \times \frac{2}{2}$

$\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$

Use 6 as the common denominator; only the second fraction needs changing. Multiply by 1 in the form of $\frac{2}{2}$, which will result in a denominator of 6. Then add the numerators; keep the denominator.

18. $\frac{4}{5} + \frac{5}{7}$

$\frac{4}{5} \times \frac{7}{7} + \frac{5}{7} \times \frac{5}{5}$

$\frac{28}{35} + \frac{25}{35}$

$\frac{53}{35} = 1\frac{18}{35}$

Use 35 as the common denominator. In this example, and many others, the common denominator is the product of the two denominators (5x7). Add the resulting numerators. The numerator is larger than the denominator, so it is an improper fraction that can be simplified.

19. $3\frac{1}{3} + 2\frac{1}{2}$

$\frac{10}{3} \times \frac{2}{2} + \frac{5}{2} \times \frac{3}{3}$

$\frac{20}{6} + \frac{15}{6} = \frac{35}{6} = 5\frac{5}{6}$

Convert the mixed numbers to improper fractions, and use six as the common denominator. Adding the numerators gives us 35, and we simplify back to a mixed number.

OR $3 + \frac{1}{3} \times \frac{2}{2} + 2 + \frac{1}{2} \times \frac{3}{3}$

$3 + \frac{2}{6} + 2 + \frac{3}{6}$

$5\frac{5}{6}$

It is not always necessary to convert to improper fractions. We can just convert the fractions. Add the whole numbers together, then add the fractions together. This example does not require simplification.

20. $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$

$\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

The denominators are the same, so we just subtract the numerators.

The result has a common factor of 2, so it simplifies to $\frac{1}{2}$.

21. $\frac{5}{12} - \frac{1}{3}$

$\frac{5}{12} - \frac{1}{3} \times \frac{4}{4}$

$\frac{5}{12} - \frac{4}{12} = \frac{1}{12}$

Use 12 as a common denominator; only the second fraction needs to change. Then subtract the numerators, keep the denominator.

22. $2 - \frac{5}{7}$

$\frac{2}{1} \times \frac{7}{7} - \frac{5}{7}$

$\frac{14}{7} - \frac{5}{7} = \frac{9}{7} = 1\frac{2}{7}$

Use 7 as the common denominator, and write 2 as the fraction $\frac{2}{1}$. Subtraction numerators. The result is improper since the numerator is greater than the denominator. Simplify to a mixed number.

23. $2\frac{5}{8} - 1\frac{2}{3}$

$2 + \frac{5}{8} \times \frac{3}{3} - 1 + \frac{2}{3} \times \frac{8}{8}$

$2 + \frac{15}{24} - 1 + \frac{16}{24}$

$1 + \frac{24}{24} + \frac{15}{24} - 1 + \frac{16}{24}$

$1 + \frac{39}{24} - 1 + \frac{16}{24} = \frac{23}{24}$

We could convert the mixed numbers first, then find a common denominator. But we can also just convert the fractions to have common denominators. When the numerator being subtracted is bigger than the numerator it is being subtracted from, convert a value of 1 as $\frac{24}{24}$.

24. $\frac{6}{7} - \frac{1}{3}$

$\frac{6}{7} \times \frac{3}{3} - \frac{1}{3} \times \frac{7}{7}$

$\frac{18}{21} - \frac{7}{21} = \frac{11}{21}$

Use 21 as the common denominator, which is the product of the two denominators. Multiply each fraction by 1 in the form of a fraction with top and bottom equal to the factor needed to result in 21. Subtract the resulting numerators and keep the denominator.