Functions

A function is a relationship between inputs and outputs. The inputs are called the domain. Every number in the domain can be put into the function and the result will be one, and only one, output. The outputs are called the range.

For example, f(x) = x + 2 is read "f of x equals x plus 2." It is a function that says that, for every valid x, there is a value f(x) that is 2 more than x. A function can designate the valid domain of input numbers. The function in this example could specify that the domain is only the integers from 1 to 10. The solution set would be a list of ordered pairs that are true with the rules of this function. Those "relations," are similar to the ordered pairs of coordinates on x- and y-axes.

In our example, if the domain is the integers from 1 to 10, then the range of f(x) = x + 2 is the integers from 3 to 12. The solution set could be listed in (domain, range) format as (1, 3), (2, 4), (3, 5), (4, 6), (5, 7), (6, 8), (7, 9), (8, 10), (9, 11), and (10, 12).

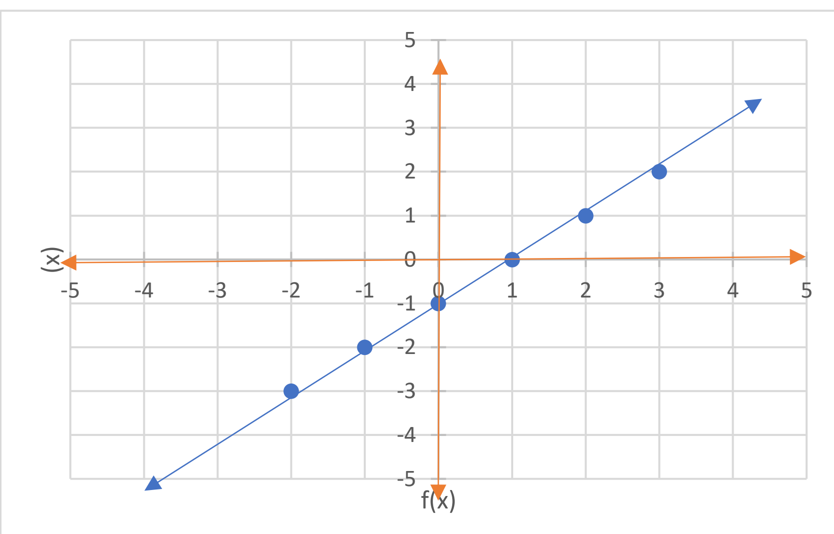
A domain might also be limited by the function itself. For example, in a function f(x) = $\frac{1}{x}$, the domain could not include the value 0. It is invalid to divide by zero, so zero is not a valid input.

A domain could also be limited by the reality of its definition. If a function calculates a value based on the number of hours in a day, the valid inputs can only be non-negative values less than or equal to 24.

Functions must return only one output for each input in the domain. That means that f(x) = $\sqrt{x}$ is not a valid function. The square root of 25 could be 5 or -5. Since there are two possible outcomes, it is not a valid function.

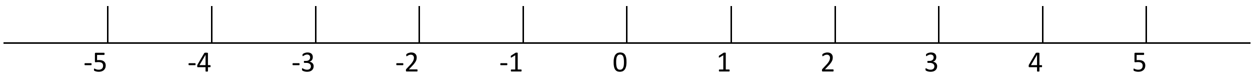
To test whether a function is valid, we can do a "vertical line test" on its graph. Any vertical line, drawn anywhere on the graph, should only pass through one point.

Here is a graph of f(x) = x – 1. Any vertical line drawn on this graph would only go through one point on the line, which confirms it a valid function. Note that the vertical axis is labeled f(x), not y. Functions are most often written as f(x), but they can also be written as g(x), pronounced "g of x," or any other letter.



Absolute Value

The absolute value of a number is its distance from zero on a number line. The symbol for absolute value is two vertical lines surrounding the value. As you can see on the number line below, 5 and -5 are the same distance away from 0. So $|-5| = 5$ says "the absolute value of negative five equals five." Since absolute value is distance, it is always a positive number.



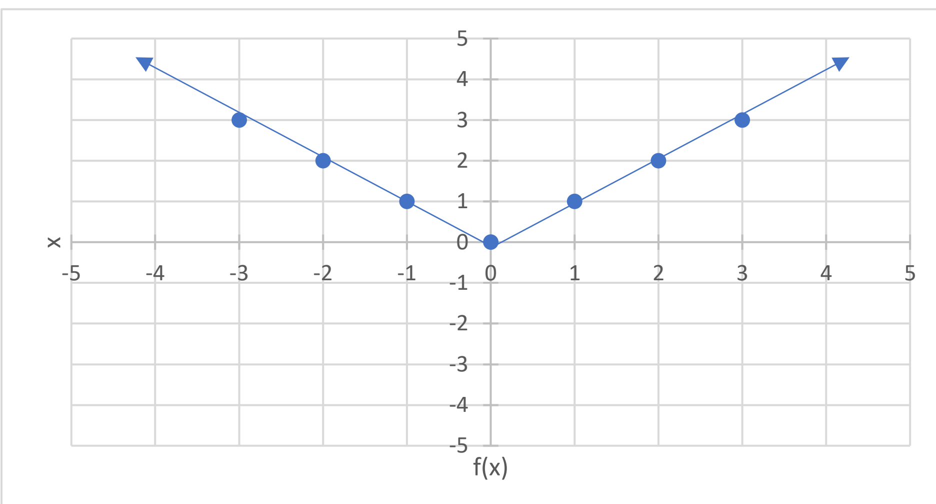
We can perform operations with absolute values.

$3 + |-7| = 10$ The absolute value of -7 = 7.

$-2\,|-3| = -6$ The absolute value of -3 is 3, which multiplied by -2 = -6.

$\frac{14}{|-2|} = 7$ The absolute value of -2 is 2; 14 ÷ 2 = 7.

Absolute value can also be written as a function and graphed. The graph below is $f(x) = |x|$. It is a function, because a vertical line anywhere on the graph would only pass through one point.



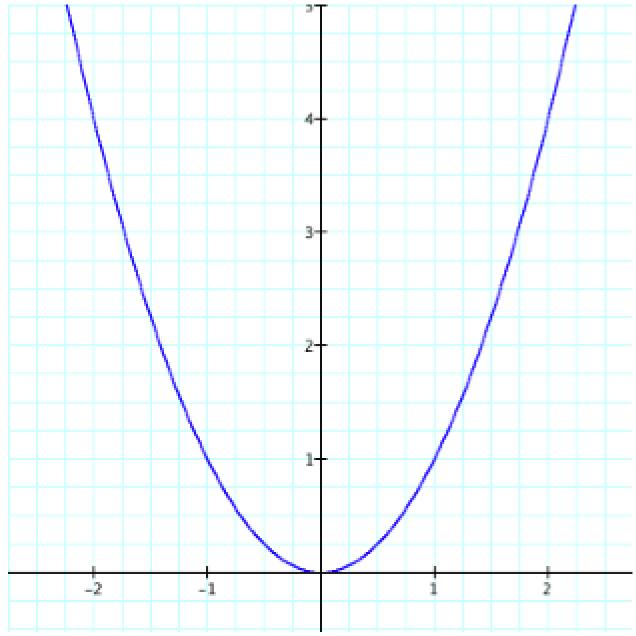
Non-Linear Equations

In Section 11, we graphed "linear" equations which are straight lines. All the variables in linear equations have exponents of one. In this section, we will introduce the basic concepts of non-linear equations. More advanced math courses cover this topic in much greater detail.

Non-linear equations have one or more variables with an exponent other than one. When graphed, they have curved lines. That means the slope of non-linear equations changes along the line.

The equation $y = x^2$ is a non-linear equation. To graph $y = x^2$, make a table of x- and y-values to plot on the graph. The more points you include, the more accurate the graph will be.

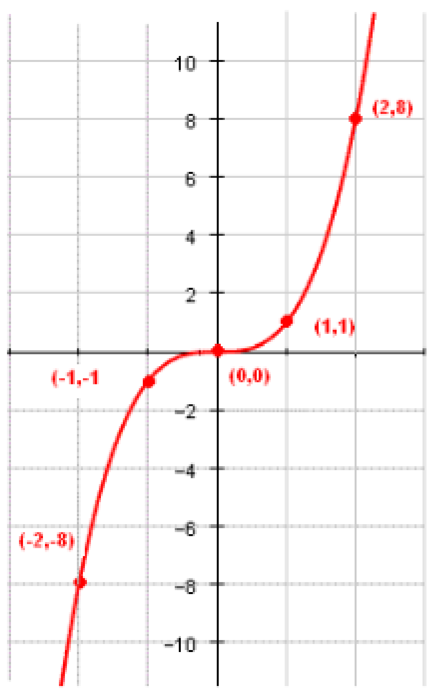
x	$y = x^2$
0	0
1	1
-1	1
2	4
-2	4



The shape of the graph $y = x^2$ is a parabola, which is a u-shaped, symmetrical curve. The vertex of this graph is at the point (0, 0). The vertex is also the lowest (or highest) point of the curve, so it is also called a "minimum" (or a "maximum").

The graph of $y = x^3$ shows an "inflection point," where the curve changes direction. Here the point (0, 0) is the inflection point.

x	$y = x^3$
0	0
1	1
-1	-1
2	8
-2	-8



Quadratic Equations

We factored simple quadratic expressions in Section 16. A quadratic equation defines y in terms of x with a squared (but no higher) exponent. The standard format of a quadratic equation is $y = ax^2 + bx + c$, where $a \neq 0$. The graphs of quadratic equations are non-linear curves that are parabolas.

Factoring a quadratic equation finds the "roots," or the points where the parabola crosses the x-axis. Since y = 0 on the x-axis, any value of x that makes y = 0 is a root.

One of the equations from Section 16 is $y = x^2 + 2x - 8$. Factoring with FOIL results in y = (x – 2)(x + 4). The factors (x – 2) and (x + 4) are multiplied together to equal y. If either one of them equals zero, the product, y, is 0. So there are two roots, or two values that will make y = 0.

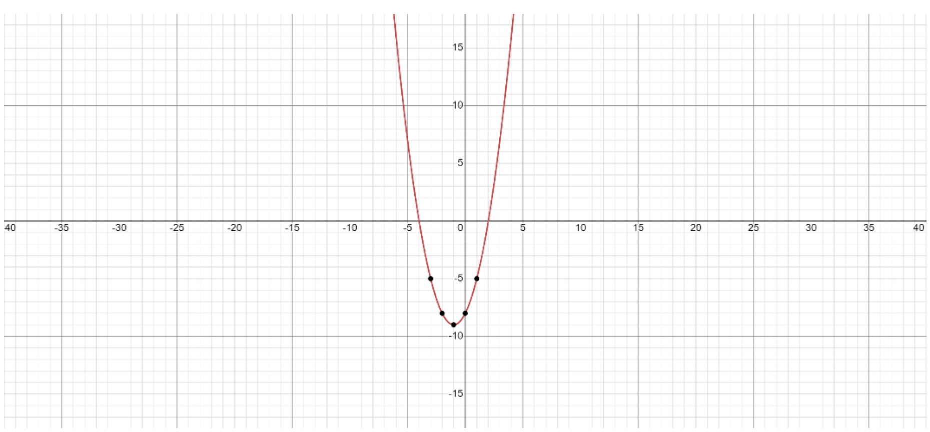
$y = x^2 + 2x - 8$
y = (x – 2) (x + 4)
(x – 2) = 0 OR (x + 4) = 0
x = 2 OR x = -4

Therefore, the graph of this equation is a parabola that crosses the x-axis at (2, 0) and (-4, 0).

There are two ways to find the x-value of the vertex. First, since parabolas are symmetrical, we know that the vertex is halfway between the roots. The average of the roots' x-values is the x-value of the vertex. In this example, that is $\frac{2+(-4)}{2}$ = -1. Substituting (-1) as the x-value in the original equation will find the vertex's y-value.

$y = x^2 + 2x - 8$
$y = (-1)^2 + 2(-1) - 8$
y = 1 -2 - 8
y = -9

Now we know that for $y = x^2 + 2x - 8$, the vertex is at (-1, -9), and the roots are (2, 0) and (-4, 0). We can calculate some other x- and y-values as needed to plot the graph:



A second way to find the x-value of the vertex is with the equation $x = \frac{-b}{2a}$. In this example that is $x = \frac{-2}{2(1)}$, or x = -1.

Not all quadratic equations can be factored easily. Instead, there is a quadratic formula that can solve any quadratic equation. It is especially helpful when factoring is difficult or impossible.

The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The values of a, b, and c are the coefficients of the quadratic equation in its standard form y = ax^2 + bx + c.

Another expression factored in Section 16 becomes y = $6x^2$ – 7x – 20 as a quadratic equation. Factored, this is y = (3x + 4) (2x – 5). To find the roots, we can set both factors equal to 0 and solve for x.

y = $6x^2$ – 7x – 20
y = (3x + 4) (2x – 5)
(3x + 4) = 0 OR (2x – 5) = 0
3x = -4 2x = 5
X = $\frac{-4}{3}$ x = $\frac{5}{2}$

The quadratic formula finds the same x-values.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(6)(-20)}}{2(6)}$$

$$x = \frac{7 \pm \sqrt{49 + 480}}{12}$$

$$x = \frac{7 \pm 23}{12}$$

X = $\frac{-16}{12} = \frac{-4}{3}$ OR X = $\frac{30}{12} = \frac{5}{2}$

These x-values are the roots of the equation y = $6x^2$ – 7x – 20.

This section is only an introduction to this topic. You can learn much more about non-linear equations in more-advanced math classes.

Practice Problems

1. For what value of x is f(x) = $\frac{x+3}{x-2}$ undefined?

2. Is f(x) = x^2 a valid function?

3. What is f(x) for x = 7: f(x) = 2x – 5

4. |-10| - |-5| =

5. 3 |-5| =

6a. What are the coordinates of the roots of y = x^2 – 2x – 24?

6b. What are the coordinates of the vertex of $y = x^2 - 2x - 24$?

7. Use the quadratic formula to find the roots of the parabola $y = x^2 + 2x - 8$.

Answer Key for Practice Problems

1. 2	2. Yes	3. 9	4. 5	5. 15
6a. (-4, 0) and (6, 0)		6b. (1, -25)	7. (0, 2) and (0, -4)	

Practice Problems Solved with Explanation

1. It is not possible to divide by zero, so a denominator of 0 would make any function undefined. When x = 2, x – 2 would equal zero.

2. The graph of $y = x^2$ is on page 3. The graph of $f(x) = x^2$ would be the same, but the y-axis would be labeled f(x). Any vertical line on the graph would pass through only one point, so it is a valid function.

3. Substitute 7 for x to solve:
f(x) = 2x – 5
f(x) = 2(7) -5
f(x) = 9

4. |-10| - |-5|	The absolute value is the distance from 0, always positive.
10 – 5	|-10|= 10 and |-5| = 5
5	
5. 3 |-5|	|-5| = 5
15	3 x 5 = 15
6a. $y = x^2 - 2x - 24$	The roots are the points where the parabola crosses the x-axis.
y = (x) (x)	To factor with FOIL, start with the parentheses and x-terms. Factors of 24 are (1, 24); (2, 12); (3, 8); and (4,6) To get negative 24, one must be positive and one must be negative.
y = (x + 4) (x – 6)	The combination that adds together to get -2 is +4 and -6.
(x + 4) = 0 OR (x - 6) = 0	Set both factors equal to zero and solve for x.
x = -4 OR x = 6	
(-4, 0) and (6, 0)	The y-value in the coordinates is zero because y = 0 on the x-axis.

6b. $y = x^2 - 2x - 24$	The standard format is $y = ax^2 + bx + c$, so a = 1, b = -2, and c = -24.
$x = \frac{-b}{2a}$	One method to find the x-value for the vertex is this formula.
$x = \frac{-(-2)}{2(1)}$	
x = 1	
$y = 1^2 - 2(1) - 24$	Substitute x = 1 into the original equation.
Y = -25	Solve for y.
(1, -25)	Write the answer in (x, y) coordinate format.
7. $y = x^2 + 2x - 8$	The standard format is $y = ax^2 + bx + c$, so a = 1, b =2, and c = -8
$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	Use the quadratic formula.
$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-8)}}{2(1)}$	Substitute the coordinates from the original equation.
$x = \frac{-2 \pm \sqrt{4+32}}{2}$	
$x = \frac{-2 \pm \sqrt{36}}{2}$	
$x = \frac{-2 \pm 6}{2}$	
x = 2 or -4	
(2, 0) (-4, 0)	Write the coordinates in (x, y) format.