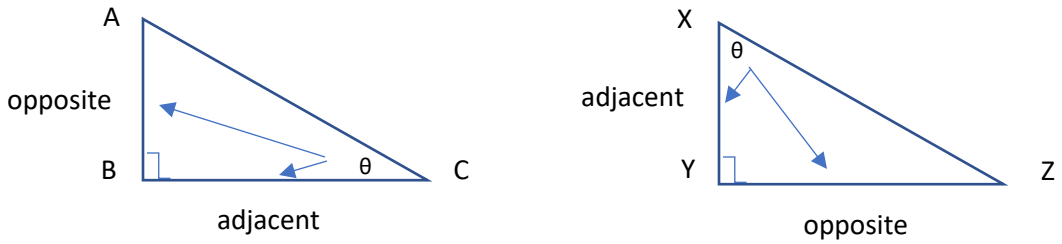
Relationships in Right Triangles

In Section 14, we saw that congruent triangles have the same angle measurements and the same side lengths. Similar triangles have the same angle measurements, but their sides are proportional. The proportions of similar right triangles give us the trigonometric functions. We can use those functions to calculate unknown angle measurements and side lengths.

The total of the measurement of a triangle's three angles is 180°. In a right triangle, one angle is a right angle measuring 90° and the other two angles are both acute angles, each measuring less than 90°.

The sides of the right triangles are referred to as "opposite," "adjacent" and "hypotenuse." The hypotenuse is always the side directly across from the 90° angle. The other sides are called "opposite" or "adjacent" depending on which acute angle we are looking at. The triangles ABC and XYZ below are similar.



In ΔABC, ∠B is the right angle. ∠C is marked with the Greek symbol θ, or "theta," which represents an angle's measurement. From this angle's orientation, side AB is opposite, and side BC is adjacent. Side AC is the hypotenuse.

In ΔXYZ, ∠Y is the right angle. ∠X is marked with the Greek symbol θ. From this angle's orientation, side YZ is opposite, and side XY is adjacent. Side XZ is the hypotenuse.

From precise side and angle measurements, we get ratios that apply to all similar right triangles.

Trigonometric Functions

Name	Abbreviation	Ratio	Reciprocal Name	Reciprocal Abbreviation	Reciprocal Ratio	Reciprocal on Calculator
sine	sin	$\frac{opposite}{hypotenuse}$	cosecant	csc	$\frac{hypotenuse}{opposite}$	$\sin^{-1}$
cosine	cos	$\frac{adjacent}{hypotenuse}$	secant	sec	$\frac{hypotenuse}{adjacent}$	$\cos^{-1}$
tangent	tan	$\frac{opposite}{adjacent}$	cotangent	cot	$\frac{adjacent}{opposite}$	$\tan^{-1}$

There are several memory devices to remember the ratios. Some people use SOH CAH TOA to stand for Sine = $\frac{Opposite}{Hypotenuse}$; Cosine = $\frac{Adjacent}{Hypotenuse}$; and Tangent = $\frac{Opposite}{Adjacent}$. Other people use the phrase "some old horse caught another horse taking oats away." You can use these or make up your own phrase.

Finding Trigonometry Values with Calculators

Before calculators, the values of trigonometric functions were looked up on tables or calculated with slide rules. Now most calculators have trigonometric functions.

To find the sine of a 30° angle on most calculators, you enter either:

sin	30	enter

or

sin	30	=

The result should be 0.5. Be sure that your calculator's mode is set to degrees. Some calculators, including the TI-30XS, also need to be set in classic mode.

To look up the value for 14(cos 60°) "fourteen times the cosine of 60°,' enter

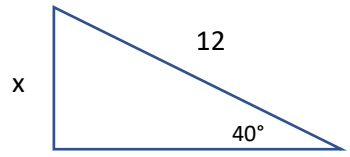
14	cos	60	enter

The result should be 7. The multiplication operation times 14 is understood because it is next to the cosine function.

Other calculator technique will be explained as needed.

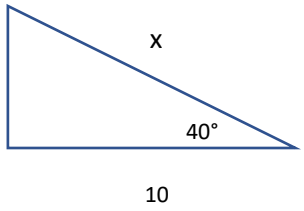
Finding Side Measurements

If you know the measurement of one angle and the length of one side, you can find the length of the other sides. Choose the sine, cosine, or tangent ratio based on what information you have and need.



In the right triangle above, find the measurement of the side labeled x to the nearest tenth. We know that the angle opposite side x is 40° and that the hypotenuse measures 12. Since we need the opposite and know the hypotenuse, we choose the sine ratio.

$\sin 40° = \frac{x}{12}$	Create an equation based on the sine ratio of $\frac{opposite}{hypotenuse}$.
12(sin 40°) = x	Multiply both sides by 12 to isolate x, by itself, on one side of the equation.
7.7 = x	On a calculator, enter: 12, sin, 40, enter. x = 7.7, rounded to the nearest tenth.



In the right triangle above, find the measurement of the side labeled x to the nearest tenth. We know the length of a side adjacent to a 40° angle, and we want to know the hypotenuse. Since we know an adjacent side and want the hypotenuse, we choose the cosine ratio.

$\cos 40° = \frac{10}{x}$ Create an equation based on the cosine ratio of $\frac{adjacent}{hypotenuse}$.

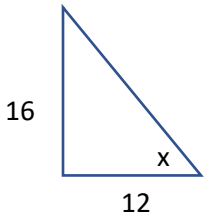
$x(\cos 40°) = 10$ Multiply both sides by x.

$x = \frac{10}{\cos 40°}$ Divide both sides by cos 40°. Now x is isolated by itself.

$x = 13.1$ On a calculator, enter: 10 ÷ cos 40, enter. Round to the nearest tenth.

Finding Angle Measurements

If you know the measurement of two sides, you can find the angle measurements. Choose the sine, cosine, or tangent ratio based on what information you have and need.



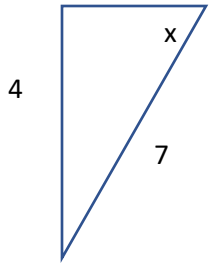
In the right triangle above, find the measurement of angle x to the nearest tenth. We know the opposite and adjacent side measurements, so we choose the tangent ratio.

$\tan x° = \frac{16}{12}$ Create an equation based on the tangent ratio of $\frac{opposite}{adjacent}$.

$x° = \frac{16}{12}\frac{1}{tan}$ Divide both sides by tangent; that's the same as multiplying by its reciprocal.

$x° = 53.1°$ On a calculator, enter: 2nd, tan, 16 ÷ 12, enter. Round to the nearest tenth.

On most calculators, division needs to be entered after the trig function. Multiplication is usually entered before the trig function, like in the previous examples.



In the right triangle above, find the measurement of angle x to the nearest tenth. We know the opposite and hypotenuse, so we choose the sine ratio.

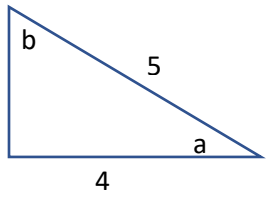
sin x° = $\frac{4}{7}$ Create an equation based on the sine ratio of $\frac{opposite}{hypotenuse}$.

x° = $\frac{4}{7}\frac{1}{sin}$ Divide both sides by sine; that's the same as multiplying by its reciprocal.

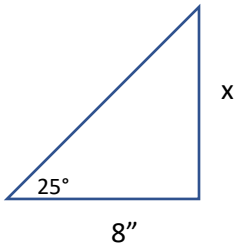
x° = 34.8° On a calculator, enter: 2^{nd}, sin, 4 ÷ 7, enter. Round to the nearest tenth.

Practice Problems

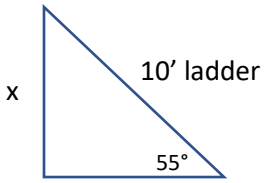
1. If this right triangle, find the measurement of $\angle a$ and ∠b to the nearest tenth. Check to be sure they add to 90°.



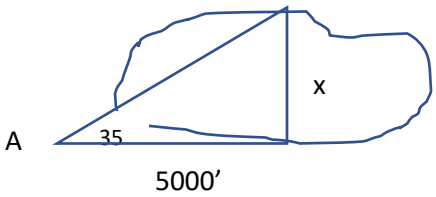
2. In this right triangle, find the length of side x to the nearest tenth.



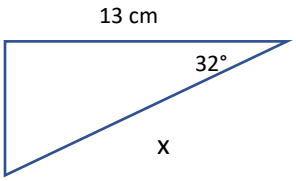
3. A 10’ ladder is placed against a building at a 55° angle. How high does it reach up the building?



4. A surveyor at Point A needs to find the distance across a lake. At his location, he is 5000' from a point on one side and 35° to a point on the other side. How wide is the lake to the nearest foot?



5. In this right triangle, find the length of side x to the nearest tenth.

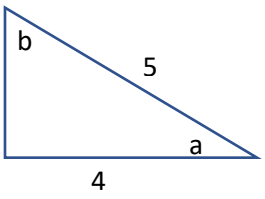


Answer Key for Practice Problems

1. $\angle a = 36.9°$ and $\angle b = 53.1°$ 2. 3.7" 3. 8.2' 4. 3501' 5. 15.3cm

Practice Problems Solved with Explanation

1.



cos a = $\frac{4}{5}$ Create an equation based on cosine ratio $\frac{adjacent}{hypotenuse}$.

a = $\frac{4}{5}\frac{1}{cos}$ Divide both sides by cos; that's the same as multiplying by its reciprocal.

a = 36.9° On a calculator, enter: 2nd, cos, 4 ÷ 5 enter. Round to nearest tenth.

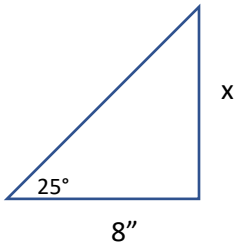
sin b = $\frac{4}{5}$ Create an equation based on sine ratio $\frac{opposite}{hypotenuse}$.

b = $\frac{4}{5}\frac{1}{sin}$ Divide both sides by sin; that's the same as multiplying by its reciprocal.

b = 53.1° On a calculator, enter: 2nd, sin, 4 ÷ 5 enter. Round to nearest tenth.

36.9° + 53.1° = 90° Add a + b to prove the two acute angles add to 90°.

2.

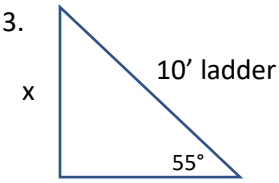


tan 25° = $\frac{x}{8}$ Create an equation based on the tangent ratio $\frac{opposite}{adjacent}$.

8 (tan 25°) = x Multiply both sides by 8.

3.7” = x On a calculator, enter: 8, tan, 25, enter. Round to nearest tenth of inch.

3.

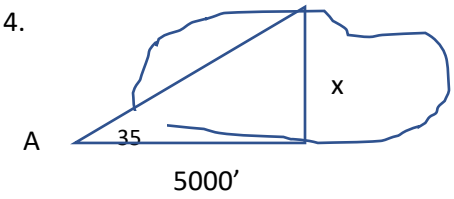


sin 55° = $\frac{x}{10}$ Create an equation based on the sine ratio $\frac{opposite}{hypotenuse}$.

10 (sin 55°) = x Multiply both sides by 10.

8.2’ = x On a calculator, enter: 10, sin, 55, enter. Round to nearest tenth of foot.

4.

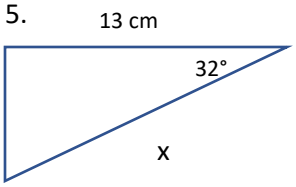


tan 35° = $\frac{x}{5000}$ Create an equation based on the tangent ratio $\frac{opposite}{adjacent}$.

5000(tan 35°) = x Multiply both sides by 5000.

3501’ = x On a calculator, enter 5000, tan, 35 enter. Round to the nearest foot.

5.



$\cos 32° = \frac{13}{x}$	Create an equation based on the cosine ratio $\frac{adjacent}{hypotenuse}$.
$x(\cos 32°) = 13$	Multiply both sides by x.
$x = \frac{13}{\cos 32°}$	Divide both sides by cos 32°.
$x = 15.3$ cm	On a calculator, enter: 13 ÷ cos 32. Round to the nearest tenth centimeter.