Place Value

Humans have ten fingers, so our number system is based on ten. Each position, or "place" is worth 10 times the value of the place to its right. A decimal point marks the spot where things become less than an entire object. Even after the decimal point, every place is 10 times the value of the place to its right.

Million	Hundred thousand	Ten thousand	Thousand	Hundred	Ten	One	Decimal Point	Tenth	Hundredth	Thousandth	Ten Thousandth
1,	0	0	0,	0	0	0	.	0	0	0	1

This chart shows the names of the places in our decimal system. The number given as an example is 1,000,000.0001. We say "one million and one ten thousandth." The commas are not really necessary, but we put a comma every three places to the left of the decimal point to make long numbers easier to read. It's common for people to say "point zero zero zero one," but it's more accurate to say "and" for the decimal point and then say the number and its place value.

Another example is the number 3,457,892.5 which is read "three million, four hundred fifty-seven thousand, eight hundred ninety-two and five tenths."

Our U.S. currency is based on the decimal system, and money is a good way to practice decimals. The amount $153.12 is read "one hundred fifty-three dollars and twelve cents."

Hundred Dollar Bill	Ten Dollar Bill	One Dollar Bill	Decimal Point	Dime	Penny
1	5	3	.	1	2

Quarters and nickels are nice, but even if our currency had only the bills and coins in this table, we could make any payment. We all know that a hundred-dollar bill is worth 10 ten-dollar bills; a ten-dollar bill is worth 10 one-dollar bills; a one-dollar bill is worth 10 dimes; and a dime is worth 10 pennies.

Thinking of money helps explain "borrowing" when we perform subtraction. The term "borrowing" isn't very accurate, since we aren't getting anything we didn't already have. It is better to think of it as "regrouping" or even "trading."

Add Decimals

For both addition and subtraction of decimals, the decimal points need to be "lined up" as if in vertical columns. That's because we are going to add the values in each place value together. Tenths need to be added to tenths, hundreds added to hundreds, etc. Any quantity of numbers can be added together, provided they are properly lined up.

Line the columns up by adding zeroes to the right side so all numbers have the same decimal places.

Once the numbers are positioned properly, add together the values in each position. If the total is 9 or less, we just write the number below the addition line in the correct place. If the total is 10 or more, write the last digit below and "carry," or bring, the other digits up to the next place value.

For example: 357.8 + 12.52 =

357.80 + 12.52	line up and add a 0: 357.80, so both have two places after decimal
357.80 + 12.52 2	Start on the right, in the hundredths position. 0 + 2 = 2
1 357.80 + 12.52 32	Next do one column to the left, the tenths. 8 + 5 = 13, so write the 3 below and the 1 above in the next position to the left.
1 1 357.80 + 12.52 0.32	Move one column left, and write the decimal place lined up. Now 1 + 7 + 2 = 10, so write the 0 below and "carry" the 1 above to the left.
1 1 357.80 + 12.52 70.32	Move one column left, now we're in the tens place, and 1 + 5 + 1 = 7. Just write the 7 in the tens place.
1 1 357.80 + 12.52 370.32	Move one column left, and there is just 3 in the hundreds place. 3 + 0 = 3

> This is the process of "carrying the one."
>
> Adding 8 + 5 gives us 13. Note that's 3 in the original column, and 1 that is worth ten times as much as the original column. That moves it over to the next column to the left. We write it on top, in line with the other numbers in its new column.

Subtract Decimals

Decimals also need to be lined up for subtraction. This keeps each value with its own place value (tenths with tenths, for example). Fill in with zeros as needed.

Once the numbers are positioned properly, subtract the number on the second line FROM the number on the top line. If the number on the second line is bigger than the number above it, we will need to "trade" or "regroup" the values on the top line.

For example: 942.57 – 34.6 =	942.57 - 34.60	line up and add a 0 to 34.6 so both have two places after decimal
	942.57 - 34.60 7	Start in the column on the right. 7 – 0 = 7, so write the 7 directly below, in its original column.
	1 15 94~~2~~.57 - 34.60 .97	Now we'd like to subtract 6 from 5, but 6 is bigger than 5. But we can trade 1 from next column to the left worth 10 in our current column. The 2 is now a 1, and we add 10 to 5 to get 15. And 15 – 6 = 9.
	3 11 15 9~~42~~.57 - 34.60 7.97	Same process, because 4 is bigger than 1. So trade 1 from the next column worth 10 in our current column. 11-4 = 7.
	3 11 15 9~~42~~.57 - 34.60 907.97	Next subtract 3 – 3 = 0. Finally 9 – 0 = 9, so the answer is 907.97. Don't forget to bring the decimal point straight down.

To summarize the trading process, always remember that, as we go each position to the left, the values are worth 10 times the values in the column to the right. Think of this example in U.S. currency. We don't have enough dimes (tenths place), so we trade one of our one-dollar bills for 10 dimes. Then we don't have enough one-dollar bills, so we trade a ten-dollar bill for 10 one-dollar bills. The trading doesn't change the value of our money, just puts it into a format that made the subtraction possible.

Multiply Decimals

Multiplication with decimals does not require lining up the decimals, and it does not require that the numbers being multiplied have the same number of decimal places.

We start with the last digit on the right of the second number, and multiply it by the entire top number, one column at a time. Like in addition, if the result if 9 or less, we just write the number in its correct column. If the result is 10 or more, we write the last digit below and again "carry" the other digits up to the next place value.

For example: 437.1 x .46 =

```
 437.1
x  .46
     6
```

To start, multiply 6 by 437.1. The first column gives us 1 x 6 = 6, so write 6 below the line.

```
 2 4
 437.1
x  .46
 26226
```

Next, 6 x 7 = 42. Write the 2 below, "carry" the 4 to the next column.
Next, 6 x 3 = 18, and we add the 4 we carried.
18 + 4 = 22. Write the 2 below, carry the other 2.
Next, 6 x 4 = 24, plus the 2 we carried = 26. No need to carry any further, just write the 26 below the line.

> Why do we put zeroes as "place holders" when we multiply?
>
> Place value!
>
> Each time we move another column to the left, the value is 10 times as much as the previous column. So one column over, insert one zero. Two columns over, insert two zeros, and so on.

```
 1 2
 437.1
x  .46
 26226
174840
```

For the next line, write one 0 as a place holder under the farthest column to the right. Multiply, carry and add as before. 4 x 1 = 4. Then 4 x 7 = 28. Write the 8 next, and "carry" the 2. 4 x 3 = 12, plus the 2 that we carried 14. Write the 4 below and carry The 1. Finally, 4 x 4 = 16, add the 1 = 17.

```
 437.1
x  .46
 26226
174840
201.066
```

After repeating this process as many times as there are digits in the second number, now we add together the results. Finally, count the total number of places to the right of the decimal in the original numbers. There is one in 437.1, and two in .46. That's a total of three, so our answer will have three places right of the decimal.

Divide Decimals

"Long" division has several steps, but, once you learn the sequence, they just repeat until you are done. First get things set up. Our example is 2.4 ÷ .08 ("two and four tenths divided by eight hundredths"). The 2.4 goes inside the division symbol, and the .08 goes "by" it (we're dividing "by" .08).

Example: .08 ⌈2.4	The next step is to move the decimal point in the divisor (the number we are dividing by) all the way to the right. In this example that's two places, so we also move the decimal point in the dividend (the number we are dividing into) two places.
. 8. ⌈240.	The decimal point then goes straight up, directly over its new place.
3 . 8. ⌈240.	Now we repeat this sequence: Divide, Multiply, Subtract, Bring Down. 8 is bigger than 2, so look at the next two digits together. 24 ÷ 8 = 3, so we write that above the 4.
3 . 8. ⌈240. - 24 00	3 x 8 = 24, which we line up with the 3 above. 24 – 24 = 0. We bring down the last digit of 240, which is 0.
30. 8. ⌈240. - 24 00 00	0 ÷ 8 = 0, so we write that above the 0 of 240. 0 x 8 = 0. 0 – 0, and there's nothing left to bring down. Once we are left with 0 and nothing more to bring down, we are done. 2.4 ÷ .08 = 30.

To summarize, set the problem up, moving the decimal points to the right as needed. Then repeat: Divide, Multiply, Subtract, Bring Down. When every number has been brought down and your subtraction result is 0, you are done.

If you have brought down each number, but you still have remainders, keep going by bringing down zeros. If the solution has more decimal places than required, round to the nearest appropriate place (see the Rounding section for more information).

Rounding Decimals

Sometimes when we divide we get more decimals after the decimal point than are meaningful, or "significant". For example, imagine you are calculating the average number of students in a class, and the result is 24.7. There's no such thing as seven-tenths of a student, so it makes sense to round to the nearest whole number (which would be 25).

The first step in rounding numbers is to determine how many digits matter to you. A test question might tell you how many digits matter, or you might use common sense like in our example with a number of students.

Calculate your answer to one place MORE than the digits of your final answer. If that number is less than 5, keep the value of your final significant digit. If, however, the "extra" position is 5 or greater, raise your final digit up by one. You do not keep that extra digit after you round.

For example: Round 256.739 to the nearest tenth. The tenths place is where the 7 currently is. So look at the next digit to the right, in this case 3. 3 is less than 5, so we keep the 7. The answer is 256.7. Notice that even though you had another digit further right, 9, it is not part of your decision.

Another example: Round 549.735 to the nearest hundredth. The hundredths place is where the 3 currently is. So look at the next digit to the right, which is a 5. 5 is greater than or equal to five, so we raise the number in the hundredths (3) place up by one (to 4). The answer is 549.74.

Practice Problems

1. Write the decimal form of "five hundred forty-three thousand five hundred and twelve thousandths."

Add the following:

2. 98.3 + 24.52   3. 897.49 + 102.51   4. 0.75 + 1.4

Subtract the following:

5. 836.2 – 23   6. 501.43 – 67.89   7. 75 – 24.99

Multiply the following:

8. 57.5 x 3   9. 8.329 x .24   10. 544 x .5

Divide the following:

11. 9.75 ÷ .25   12. 463.36 ÷ 18.1   13. 85.488 ÷ 16.44

14. Round 407.32 to the nearest tenth   15. Round 637.499 to the nearest hundredth

16. Round 406.507 to the nearest ones

Answer Key for Practice Problems

1. 543,500.012		2. 122.82	3. 1000	4. 2.15	5. 813.2
6. 433.54	7. 50.01	8. 172.5	9. 1.99896	10. 272	11. 39
12. 25.6	13. 5.2	14. 407.3	15. 637.50	16. 407	

Practice Problems Solved with Explanation

1. "Five hundred forty-three thousand five hundred and twelve thousandths" is 543,500.012. Remember that the "and" indicates the decimal point; then there must be a zero to indicate that the twelve is thousandths (not hundredths.)

```
      1
2.   98.30
   + 24.52
    122.82
```

Be sure the decimal places are lined up, and add a 0 in the hundredths place of 98.30. Note that in the ones place, 8+4 = 12, so the 2 goes below and the 1 carries to the next place value. 1 + 9 + 2 = 12.

```
      1 1 1 1
3.   897.49
   + 102.51
    1000.00
```

Every column adds up to 10, which means we put the 0 below, and the 1 carries to the next place value. Remember, 10 in any column can be traded for 1 in the next column to the left.

```
     1
4.     .75
   + 1.40
     2.15
```

First the decimal places are lined up by adding a 0 to the hundredths place of 1.4. In the hundredths column, 7 + 4 = 11, so we write the 1 in that column and carry the 1 to the ones column.

```
5. 836.2
   - 23.0
   813.2
```

No "trading" or regrouping is needed for this problem. Starting from the right, subtract within each column, write the answer below, then move to the next column to the left and repeat.

```
   4 9 10 13 13
6.  501.43
   -  67.89
    433.54
```

Every place needs to be regrouped for this problem. Start by trading one from the tenths place for 10 in the hundredths place; there are already 3 hundredths, so that makes 13. Notice that when we get to the ones place, there are no tens to regroup. So we go another place over, and the 5 hundreds become 4 hundreds, 9 tens and 10 ones.

```
      4  9 10
7.   75.00
    - 24.99
     50.01
```

We have to go two places to the left (ones) to make a trade for hundredths. We regrouped 5 ones as 4 ones + 9 tenths + 10 hundredths. And of course, we lined up the decimal places and added those zeros first!

8.
```
  2 1
  57.5
 x   3
 172.5
```
No need to add additional decimal places to multiply. Start in the column farthest left. 3 x 5 = 15, so the 5 goes down, and the 1 carries. Next, 3 x 7 = 21, then add the 1 that we carried and get 22. The 2 goes below the line, and the 2 carries to the next place. 3 x 5 = 15 and add the 2 for 17. There is one place after the decimal in 57.5 and NO places after the decimal in 3, so our answer has a total of one place after the decimal point.

9.
```
   8.329
    x .24
   33316
  166580
  1.99896
```
Ignore decimal places for a moment, and we multiply 8329 by 4, and get 33316. Put a zero as a place holder in the farthest column to the right first, then multiply 8329 x 2, getting 16658. That's the last digit to multiply, so now add the results. The starting numbers have a total of five places after the decimal, so our ending result does too.

10.
```
 544
x  .5
 272.0
```
There is a total of one place after the decimal point in the original numbers, so the final result also has one decimal point. Notice, though, that we can write the answer as 272, since just zeros after the decimal point are not necessary.

11.
```
        39.
25. ⌈975.
     75
     225
     225
       0
```
Notice first that, although we started with .25, we do not want any decimal places in the divisor. We move that decimal place two places to the right, and so we must also do the same with 9.75, which becomes 975. Repeat the steps: divide, multiply, subtract, bring down until all numbers are brought down and the result of subtracting is 0.

12.
```
          25.6
181 ⌈4633.6
      362
      1013
       905
       1086
       1086
          0
```
The decimal point in 18.1 was moved one place to the right, so the same results in 463.36 becoming 4633.6. 181 goes into 463 2 times; 2 x 181 = 362; 463 – 362 = 101; bring down the 3. 181 goes into 1013 5 times; 5 x 181 = 905; 1013 – 905 = 108; bring down the 6. 181 goes into 1086 6 times; 6 x 181 = 1086; 1086 – 1086 = 0. Remember that the decimal point goes straight up from the dividend (4633.6) into the quotient (25.6).

13. 5.2
1644 ⎾8548.8
8220
3288
3288
0

Move the decimal two places to the right in both 16.44 and 85.488.
We repeat divide, multiply, subtract, bring down until done.

14. 407.32 → 407.3

We want to round to the nearest tenth. There is a 3 in the tenths place, Look one digit to the right, at the 2 in the hundredths place. That Is less than 5, so the tenths place remains a 3.

15. 637.499 → 637.50

We want to round to the nearest hundredth. There is a 9 in the hundredths place. Look one digit to the right, at the 9 in the thousandths place. That is greater than 5, so the hundredths place rounds up to 10. That affects the tenths place too, changing the 4 to 5.

16. 406.507 → 407

We want to round to the nearest ones. There is a 6 in the ones place. Look one digit to the right, at the 5 in the tenths place. That is greater than or equal to 5, so the ones place rounds up to 7.