An equation has an "equals sign" (=). That means that whatever is on the left has the same value as (is equal to) whatever is on the right. For example, "3 + 2 = 5" is an equation.

An "equation" is different from an "expression," because an expression does not have an equals sign. "$3X^2 + 4$" is an expression, but not an equation.

Equations may contain "variables," such as "x" or "y." A variable can be any letter or symbol which stands for a specific value (or values) that is unknown. We solve equations to find the value of the variable(s), making known what was unknown. Therefore, the solution should look something like "x = 4." That means that the value of 4 is what make the equation "true."

Treat Both Sides the Same

It's important to remember that both sides of an equations have the same (equal) value. So we can "do" the same thing to both sides and they will still be equal. We can add, subtract, multiply or divide both sides of an equation by the same value and the equation will still be true.

For example: x + 7 = 12. That means that there is some unknown number, represented by x, and when that number is added to 7, the result is 12. Our goal is to find out what that unknown number is.

If we subtract 7 from both sides of the equation, we will have:

x + 7 = 12	→ this line is the original problem
<u> - 7 - 7</u>	→ this line shows a subtraction of 7 from both sides; notice how it is lined up
x = 5	→ this line shows the results; on the left x + 7 – 7 = x; on the right 12 – 7 = 5

If the process for the left side isn't clear at first, think of it this way: x + (7 – 7) = x. Adding parentheses around (7 – 7) is okay (using the associative property). Of course, (7 – 7) = 0; and x + 0 = x (additive identity property). Review Section 1 on Basic Rules if needed.

So, the solution to this problem is that X = 5. We can confirm this is right because indeed when 5 is added to 7, the result is 12.

It wasn't a random decision to subtract 7 from both sides. The goal of solving the equation is to "isolate" the variable, meaning get it all alone on one side of the equals sign. So to solve an equation, we "undo" what is being "done" to the variable. In this case, a number was being added to X. The opposite of addition is subtraction; if 7 is being added to x, we should subtract 7.

> Addition and Subtraction are Opposites
>
> Multiplication and Division are Opposites

What if the equation involves both addition/subtraction and multiplication/division? What do we undo first? Generally, we will undo addition or subtraction first, then undo the multiplication or division. For

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example: 3x + 4 = 19. This means that there is some number, represented by x, and when it is multiplied by 3 and then added to 4, the result is 19. To solve the equation, we will do the opposite.

$3x + 4 = 19$	
$-4 \quad -4$	First we subtract 4 from both sides.
$3x \quad = 15$	The result is 3x = 15; notice that we are keeping things "lined up" vertically.
$\div 3 \quad \div 3$	Next we divide both sides by 3.
$x \quad = 5$	The answer is that x = 5.

To solve this equation, we first "undo" the addition of 4 with the subtraction of 4, because addition and subtraction are opposites. Then we "undo" the multiplication by 3 with division by 3, because multiplication and division are opposites.

Here's another example: $\frac{x}{2} - 3 = 7$. There is some number x, and when it is divided by 2 and then 3 is subtracted from it, the result is 7. Again, we will do the opposites.

$\frac{x}{2} - 3 = 7$	
$+3 \quad +3$	First we will add 3 to both sides. That is the opposite of subtracting 3.
$\frac{x}{2} \quad = 10$	The result is $\frac{x}{2} = 10$; we are keeping things "lined up" vertically.
$*2 \quad *2$	Next we multiply both sides by 2 (using * for multiplication avoids confusion
$x \quad = 20$	with the variable "x"). The answer is that x = 20.

As long as we always carefully do the same operation to both sides, we will still have a true equation. However, not every sequence of operations will produce a result in the fewest number of steps, or with the least amount of work. What if, in the problem above, we undo the division first? Will we still get the right answer? Yes, but with some extra work.

$\frac{x}{2} - 3 = 7$	
$2(\frac{x}{2} - 3) = 2*7$	First we multiply by 2, but we have to distribute the 2 over all terms.
$x - 6 = 14$	The result is x – 6 = 14.
$+6 \quad +6$	Next we add 6.
$x \quad = 20$	The result is still x = 20.

We get the same answer in the same number of steps, but we had to do a little more work distributing the multiplication across both terms on the left side. There's nothing wrong with that approach, but it took some extra work which can lead to extra mistakes or make our mistakes more difficult to spot and correct. With some practice and experience, you will know what to do first.

Combine Like-terms

Remember that "terms" are the pieces of an equation that are separated by addition and subtraction. So, in an expression like $4x^3 + 17x^2 – 32 + 8xy$, there are four terms. They are: $4x^3$, $17x^2$, -32, and 8xy.

"Like-terms" have the same variables with the same exponents. So, $4x^2$ and $9x^2$ are like-terms; their variables are "alike" since they are both x^2. They have different "coefficients" (in this case the 4 and the 9). Another example of "like-terms" would be 5xy and 12xy, because they have the same variables, or the "xy" part of the term. However, $5x^2y$ and 5xy are *not* like-terms. In that example, even though the variables and the coefficients are the same, the exponents are not the same. Even if you aren't ready to solve equations with higher-level exponents yet, you can already understand the concept for combining like-terms.

When we are solving equations, combining like-terms is almost always a good first strategy. Consider this example: 3x + 3 + 9x = 27. There are two "x" terms: 3x and 9x. Before we do any operations to solve the equation, we should combine those terms. How do we combine them? In this example they are both positive (they both have addition in front of them, even if the first term doesn't explicitly have a "+" sign in front of it). So, we can just add them together: 3x + 9x = 12x. We don't know what x stands for yet, but we know that it is the same "thing" in both places. And, of course, 3 things + 9 things = 12 things, regardless of what the "things" are. So:

3x + 3 + 9x = 27	
12x + 3 = 27	We combine the like-terms, 3x + 9x = 12x
-3 -3	Subtract 3 from both sides (that's also combining the like-terms 3 and 27!).
12x = 24	
÷12 ÷12	Divide both sides by 12.
x = 2	

Here's another example where we have "like-terms" on both sides of the equation. We will combine them so that the variable will only be on one side of the equation.

2x + 15 = 8x -9	
-2x -2x	Combine like-terms by subtracting 2x from both sides.
15 = 6x -9	
+9 +9	Add 9 to *"undo" the operation that is on the same side as the variable!*
24= 6x	
÷6 ÷6	Divide both sides by 6.
4 = x	

There's a couple of interesting things to learn from this example. First, notice that our answer reads "4 = x." That is, of course, the same as "x = 4." It is absolutely okay to have our variable on either side of the equals sign!

Second, when we combine like-terms, we subtract the term with the smaller coefficient. In other words, we subtract 2x instead of subtracting 8x. Does that matter? No, of course not, as long as we treat both sides the same, we can do anything we want. But it does avoid working with negative numbers. Look at it the other way:

2x + 15 = 8x – 9	
-8x -8x	This time we subtract 8x instead of 2x.
-6x + 15 = -9	
- 15 -15	Subtract 15: Undo what is on the same side of the equation as the variable.
-6x = -24	
÷-6 ÷-6	Divide both sides by -6.
x = 4	The answer is the same as before, of course!

Check Your Answer

How can you be sure your answer is right? First use the value you got as your answer and substitute it into the original problem everywhere we see the variable. Then complete the operations. If we get a "true" answer, then we must have the correct value. An equation is "true" if the left side equals the right side. Consider the last example, where the answer is x = 4.

2x + 15 = 8x - 9	Here's the original equation.
2(4) + 15 = 8(4) – 9	Everywhere there was an "x," now there's a 4 (our solution is x = 4).
8 + 15 = 32 – 9	Following PEMDAS, we perform multiplication first, then addition/subtraction.
23 = 23	The result is true, because we have the same value on the right and left.

If you follow this procedure and you get a result that isn't true (such as 3 = 4), then either your original answer is wrong, or possibly you made a calculation error when you checked your work.

Practice Problems

Solve each equation and check your work:

1. 15 = x + 5
2. 5x – 7 = 18
3. $\frac{3x}{2}$ + 4 = 10
4. 7y + 8y = 15
5. 3x + 8 = 2
6. 9x + 3 = 10x
7. 16 + 4x = 6x + 2

Answer Key for Practice Problems

1. x = 10
2. x = 5
3. x = 4
4. y = 1
5. x = -2
6. x = 3
7. x = 7

Practice Problems Solved with Explanation

1. $15 = x + 5$	Our variable, x, is being added to 5. So we "undo" by subtracting 5.
$\underline{-5 \quad -5}$	Subtract 5 from both sides.
$10 = x$	Answer is 10 = x, which is the same as x = 10.
$15 = (10) + 5$	Substitute the answer, 10, for the variable in the original problem.
$15 = 15$	15 = 15, which is true, so our answer is correct.
2. $5x - 7 = 18$	
$\underline{\quad +7 \quad +7}$	Undo the subtraction first by adding 7 to both sides.
$5x \quad = 25$	
$\underline{\div 5 \quad \div 5}$	Undo the multiplication by dividing both sides by 5.
$x \quad = 5$	
$5(5) - 7 = 18$	Substitute 5 into the original equation for x.
$25 - 7 = 18$	
$18 = 18$	The result is true.
3. $\frac{3x}{2} + 4 = 10$	
$\underline{\quad -4 \quad -4}$	First undo the addition by subtracting 4 from both sides.
$\frac{3x}{2} \quad = 6$	
$\underline{*2 \quad *2}$	Now eliminate the fraction (division) by multiplying both sides by 2.
$3x \quad = 12$	
$\underline{\div 3 \quad \div 3}$	Divide both sides by 3.
$x \quad = 4$	We could combine the last two steps into one step by dividing both sides by $\frac{3}{2}$, and we would still get the same answer!
$\frac{3(4)}{2} + 4 = 10$	Substitute our answer 4 for the variable x.
$\frac{12}{2} + 4 = 10$	3 x 4 = 12.
$6 + 4 \quad = 10$	12 ÷ 2 = 6.
$10 \quad = 10$	Our answer is true.
4. $7y + 8y = 15$	This time the variable is a "y."
$15y \quad = 15$	Combine the like-terms. 7y + 8y = 15y.
$\underline{\div 15 \quad \div 15}$	Divide both sides by 15.
$y \quad = 1$	
$7(1) + 8(1) = 15$	Substitute or "plug" in 1 every place the variable appears.
$7 \quad + 8 \quad = 15$	Multiply.
$15 \quad = 15$	Add. Again, our answer is true!

5. 3x + 8 = 2

- 8 -8	Subtract 8. Yes, we will get a negative number. But we want to undo the operations that are *on the same side of the equation as the variable*!
3x = -6	
÷3 ÷3	Divide both side by 3.
x = -2	
3(-2) + 8 = 2	Substitute the solution, -2, for the variable.
-6 + 8 = 2	Multiply first.
2 = 2	Then add, and the solution is true.

6. 9x + 3 = 10x

-9x -9x	First, combine like-terms by subtracting the smaller one.
3 = x	10x – 9x = 1x, which is the same as x.
9(3) + 3 = 10*3	Substitute 3 for x everywhere in the original equation.
27 + 3 = 30	Multiply.
30 = 30	Add.

7. 16 + 4x = 6x +2

-4x -4x	Combine like-terms by subtracting the smaller 4x from both sides.
16 = 2x +2	
-2 -2	Undo the addition by subtracting 2.
14 = 2x	
÷2 ÷2	Undo the multiplication by dividing by 2.
7 = x	
16 + 4(7) = 6(7) + 2	Substitute our answer, 7, for x in the original equation.
16 + 28 = 42 + 2	Multiply first, following PEMDAS.
44 = 44	

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