A proportion describes two things that have a fixed relationship. There are two kinds of fixed relationships, "direct" and "indirect." In a direct proportion, if one increases, the other increases. In an indirect proportion relationship, if one increases, the other decreases.

Direct proportions are represented by a ratio, where $\frac{item\ 1}{item\ 2} = constant\ x$.

Indirect proportions are represented by a product, where (item 1)(item 2) = constant x.

For an example of a direct proportion, think of a situation where every 8 children must have 1 teacher. We can say that the ratio of children to teachers is 8:1 (pronounced "eight to one"). The ratio alone doesn't tell us how many children or teachers there are in the group, or even how many total people there are. But if we know one or more of those facts, we can use the proportion to find the rest of the details.

For an example of an indirect proportion, think of a job that will take a group of 4 people 6 hours. The product of people and hours gives us (4 people) times (6 hours) = 24. If we have fewer people, the job will take more time, and if we have more people, the job will take less time, but (if they all work at the same rate) the relationship stays the same.

Direct Proportion Set Up

A direct proportion is an equation where both sides are equal ratios. The ratios are usually shown in fraction format. In our example, where the ratio of children to teachers is 8:1, the fraction format is $\frac{children}{teachers} = \frac{8}{1}$.

The first step is to determine what two things have a relationship. In this example it is a number of children per teacher. Other examples could be miles per hour, dollars per gallon, cost per pizza, wages per hour, calories per serving, or students per classroom.

The next step is to write the relationship in fraction format, putting one in the numerator and the other in the denominator, like $\frac{children}{teachers}$. Then write the information you know about the relationship, staying consistent with the format you chose: the information about whatever you put in the numerator stays in the numerator. So our example will be: $\frac{children}{teachers} = \frac{8}{1}$.

Next, we usually know another piece of information or have another situation where we can apply this relationship. In our example, if we know there will be 32 children in the group, now we can also find the number of teachers. Remember to stay consistent with what's in the numerator and denominator.

Our example is now $\frac{children}{teachers} = \frac{8}{1} = \frac{32}{x}$, where x is the number of teachers. Before we solve for x, notice that once we established the relationship with children in the numerator, we keep the information about children in the numerator. It does not matter whether we start with $\frac{children}{teachers}$ or $\frac{teachers}{children}$. All that matters is that we stay consistent.

Notice also that there is an equals sign between the proportions. That means that what is on the left has the same value as what is on the right.

Solve with Cross-Multiplication

To solve the equation, we use a process called "cross-multiplication." We will multiply together each numerator by the denominator diagonally across from it, in a cross-shaped pattern. With letters, it looks like this: $\frac{a}{b} \searrow = \swarrow \frac{c}{d}$

And the result is: bc = ad

In our example, our proportion equation is: $\frac{children}{teachers} = \frac{8}{1} \searrow = \swarrow \frac{32}{x}$. When we cross-multiply, we get:

32 = 8x then divide both sides by 8:

4 = x

So in a group where the ratio of children to teachers is 8 to 1, if there are 32 children in the group, there are 4 teachers.

For another example, if a car gets 30 miles per gallon, how many miles can it go with 12 gallons? The relationship is between miles and gallons, so we write $\frac{miles}{gallons} = \frac{30}{1} = \frac{x}{12}$. Once we decide that we will put miles in the numerator and gallons in the denominator, we keep that pattern all through the setup. To solve the equation with cross-multiplication, we get: $\frac{miles}{gallons} = \frac{30}{1} \searrow = \swarrow \frac{x}{12}$

x = 360.

We can travel 360 miles with 12 gallons in a car that gets 30 miles per gallon.

Cross-multiplication is only appropriate when we have an equation with a proportion on each side. It's a common mistake to confuse cross-multiplication with multiplication of fractions. Remember, to multiply fractions, we multiply straight across. $\frac{1}{3} * \frac{3}{4} = \frac{3}{12}$.

Why does cross-multiplication work? It's really just a little shortcut. We are actually comparing fractions by finding a common denominator. If you are interested, you can apply what you know about common denominators to prove this to yourself.

Indirect Proportion Set-Up

There are many direct relationships involving many kinds of items. Indirect relationships often involve time or how long an activity takes.

In the example where it takes four people six hours to do a job, we set up the proportion as a product of the number of people and the number of hours. (people) x (hours) = job, so in this example, (4 people)(6 hours) = 24. It will take 24 total hours of work to do the job. So, if there are eight people to do the job, the relationship and the product stays the same, and now there are (8 people)(x hours) = 24. Dividing both sides by 8 solves for x, so x = 3, which means eight people can do the job in three hours.

Practice Problems

1. Solve for x: $\frac{3}{10} = \frac{x}{60}$ 2. Solve for x: $\frac{5}{x} = \frac{15}{30}$

3. If Joe earns $10 per hour, how much does he earn in 8 hours?

4. If the ratio of students to teachers is 16:1, how many students are taught by 3 teachers?

5. Mary wants to make lemonade that is 10% real lemon juice and the rest is water. How many ounces of lemon juice does she need per 60-ounce pitcher?

6. It takes Bill 3 hours to run 12 miles. How many miles does he run per hour?

7. It takes six people a total of four hours to mow the school's grass. How long would it take eight people to mow the same grass?

Answer Key for Practice Problems

1. x = 18 2. X = 10 3. $80 4. 48 students 5. 6 ounces 6. 4 miles

7. 3 hours

Practice Problems solved with Explanation

1. $\frac{3}{10} = \frac{x}{60}$ Cross-multiply numerators with the opposite denominators.

$10x = 180$

$\underline{\div 10 \quad \div 10}$ Undo multiplication; divide both sides by 10.

$x = 18$

2. $\frac{5}{x} = \frac{15}{30}$ Cross-multiply numerators with the opposite denominators.

$15x = 150$

$\underline{\div 15 \quad \div 15}$ Divide both sides by 15.

$x = 10$

3. $\frac{dollars}{hour} = \frac{10}{1} = \frac{x}{8}$ The relationship is between dollars and hours; $10 per 1 hour; $x per 8 hours. It's a direct relationship because more hours means more dollars.

$x = 80$ Cross-multiply.

4. $\frac{students}{teachers} = \frac{16}{1} = \frac{x}{3}$ The relationship is between students and teachers; 16 students per 1 teacher; x students per 3 teachers. More students require more teachers so it's a direct relationship.

$x = 48$ Cross-multiply.

5. $\frac{lemon}{total} = \frac{10}{100} = \frac{x}{60}$

$100x = 600$
$\underline{\div 100 \quad \div 100}$
$x = 6$

The relationship is between lemon juice and total lemonade. More lemonade Requires more lemons, so it's a direct proportion. Remember that 10% means 10 per hundred, which is $\frac{10}{100}$, so x ounces per 60 total ounces. Cross-multiply, then divide both sides by 100.

6. $\frac{hours}{miles} = \frac{3}{12} = \frac{1}{x}$

$12 = 3x$
$\underline{\div 3 \quad \div 3}$
$4 = x$

The relationship is hours per miles; 3 hours per 12 miles; 1 hour per x miles. Cross-multiply. Divide both sides by 3.

7. people(hours) = job

$6(4) = 8(x)$
$24 = 8x$
$\underline{\div 8 \qquad \div 8}$
$3 = x$

This is an indirect relationship because more workers can get the job done in less time. The relationship is (people) times (hours). We are told that six people do the job in four hours. Since it is the same (equal) job, we can make an equation. The other side is eight people in x hours. Divide both sides by 8.