An inequality is a lot like an equation, but there is one big difference. Equations have an equals sign, meaning that both the right and left side have the same value. Inequalities have "greater than" or "less than" signs, indicating whether the right or left side has the larger value.

The symbol ">" means "greater than," and "<" means "less than." The "greater than" sign points to the right, the direction that number lines increase. The "less than" sign points to the left, the direction that number lines decrease. Some examples of inequalities are "8>5" (8 is greater than 5), and "3<4" (3 is less than 4). With a variable, "x>10" means all the values larger than 10 make the inequality true.

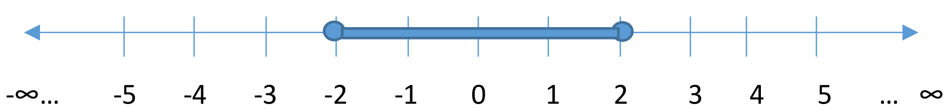
The signs can also be combined with equality: "≥" means "greater than or equal to," and "≤" means "less than or equal to." For example, the inequality "x≥10" means that x can be all values equal to 10 and larger than 10.

Inequalities on a Number Line

Inequalities can be plotted on a number line as visual solutions. The endpoint, the point where the range of included numbers begins, and it is represented by an open circle for "greater than" and "less than" solutions. For solutions that include an "or equal to" point, the endpoint is a solid circle. So, if the endpoint is filled in, the solution includes that value. The portion of the number line included in the solution is also darkened. For example:

$x \geq 2$ $x > 2$ $x \leq 2$ $x < 2$

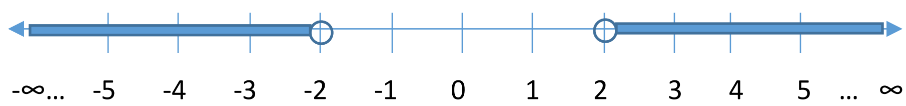
Inequalities can have compound solutions when two inequalities are joined together with "and." This joining is called the intersection of the two inequalities. Consider this example:
"$x \geq -2$ and $x \leq 2$." When the inequalities are graphed on their own, the solutions continue up or down to infinity. But when they are joined with "and," the solution will be only the intersection (where the individual inequalities overlap). The solution looks like this on a number line:



X ≥ -2 and x ≤ 2

(This can also be written as $-2 \leq x \leq 2$.)

Inequalities can also be joined together with "or," making them the union of the two inequalities. Consider the example: "x < -2 or x > 2." In this example, either part of the solution can be true, so the solution will be the union of both. The solution looks like this on a number line:



x < -2 or x > 2

When comparing these two examples, be sure to notice that the first example joins the two with "and," making it the "intersection" of the solutions. The second example joins the two inequalities with "or," making it the "union" of the solutions.

The first example has its endpoints filled in, indicating that they are included in the solution. That is because each inequality in that example includes the option for the endpoints to be "equal to" in addition to "greater than" or "less than." The second example does NOT have its endpoints filled in. In that example, the endpoints are not included in the solution because each inequality includes either "greater than" or "less than" but no option for the endpoints to be "equal to." Also, in the second example the solution continues on to infinity (∞) to the right, and negative infinity (-∞) to the left.

Solving Inequalities

Solving inequalities with variables is almost exactly like solving equations. Just like with equations, both sides of the inequality must always be treated the same. The goal is to isolate the variable by itself on one side of the inequality sign. As always, addition "undoes" subtraction; multiplication "undoes" division. And combining like-terms is generally a good idea.

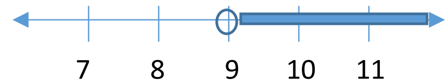
The difference is that there are two situations when the direction of the inequality sign must be reversed. The first situation is when we multiply or divide by a negative number. The second situation is when we move the variable from one side of the inequality to the other. When we do either of these two things, we reverse the sign: "greater than" changes to "less than," or "less than" changes to "greater than."

For an example of the first situation, consider the inequality "-x > 3," or "negative x is greater than 3." Think of that as "the opposite of x is greater than 3." When both sides of that inequality are divided by negative 1, the result is "x < -3." That is because, if the opposite of x is greater than 3, then x itself must be less than -3. It may be easier to understand that with numbers. So, "-1 > -3" (negative 1 is greater than negative 3) is true. When we divide both sides by -1, we get "1 < 3," (one is less than 3) which is also true. The sign changes from "greater than" to "less than."

An example of the second situation is changing "5 < x" to "x > 5." To switch x from one side of the inequality to the other, the signs must be reversed. Again, consider that example with numbers, and we see that "5 < 7" is the same as "7 > 5." It is often easier to visualize a solution and plot it on a number line when the variable is on the left.

Here are some examples of solving inequalities:

x + 7 > 16
-7 -7 — Subtract 7 from both sides.
x > 9



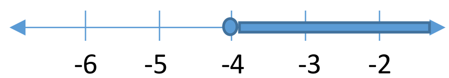
The solution is x > 9 ("x is greater than 9"). It looks like this on the number line. Notice that 9 is not a solution, but every value larger than 9 is part of the solution. To check your work, choose any value in the solution (again, not 9!). If we choose 10:

x + 7 > 16	Here's the original inequality.
10 + 7 >16	Substitute 10 for x: everywhere there was an "x," now there's a 10.
17 >16	The result is true; 17 is greater than 16.

For another example:

14 – 3x ≤ 26	
-14 -14	Undo what is on the same side as the variable: subtract 14 from both sides.
-3x ≤ 12	
÷ -3 ÷ -3	Divide both sides by -3. We divided by a negative number, so we have to reverse the sign from ≤ to ≥. The solution is "x is greater than or equal to -4."
x ≥ -4	

On the number line, the solution looks like this:

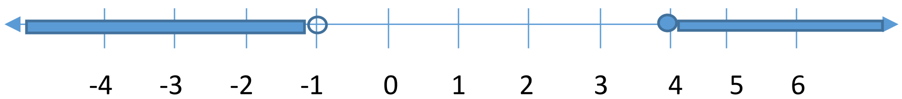


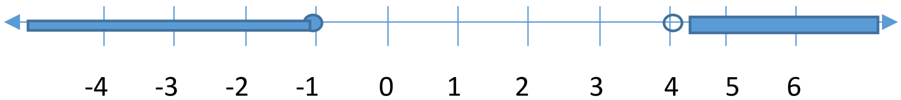
In this case, -4 is part of the solution, so we can use -4 to check the solution.

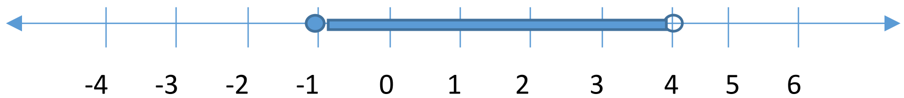
14 – 3x ≤ 26	Rewrite the original inequality.
14 – 3(-4) ≤ 26	Substitute -4 for x.
14 + 12 ≤ 26	Follow PEMDAS, multiply -3 by -4, which is 12. (signs are same; answer positive)
26 ≤ 26	Finish with addition. 26 is less than or equal to 26, so the solution is true.

Practice Problems

1. Show x ≥ 7 on a number line.
2. Show -5 < x on a number line.
3. Show -3 ≤ x < 2 on a number line.
4. Show union of x ≥ 2 or x < 1 on a number line.
5. Solve x – 3 ≤ 4.
6. Solve 3x + 5 > x – 1
7. 5x – 2 ≤ 12 – 2x
8. Solve 9 – 3x > 15
9. Which of the following is a graph of the inequality -1 ≤ x < 4?

a.


b.


c.


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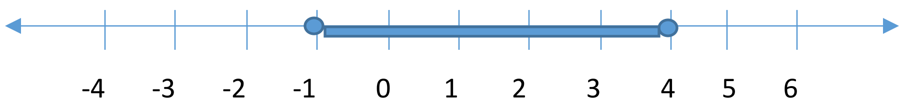
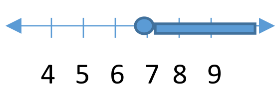
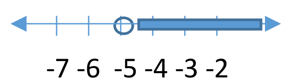
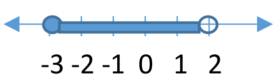
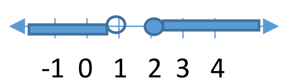


Answer Key for Practice Problems

1.


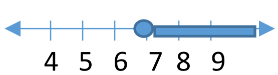
2.


3.


4.


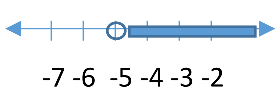
5. x ≤ 7 6. x > -3 7. x ≤ 2 8. x < -2 9. c.

Practice Problems Solved with Explanation

1.


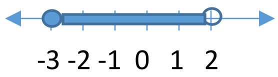
x ≥ 7

The endpoint is solid, because there is an "equal to" option. The upper end of the number line is shaded to represent that the solution includes all numbers "greater than" 7.

2.


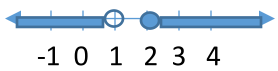
-5 < x To reverse this inequality, so the variable is on the left, the sign is also reverse. That changes it to x > -5.

The endpoint is not solid, because there is no option to the "equal to." The upper end of the number line is shaded because x > -5.

3. 

$-3 \le x < 2$

In this example, both $-3 \le x$ AND $x < 2$ must be true. That means it is the intersection of both conditions. So, the endpoint at -3 must be solid because x can be equal to -3. The endpoint at 2 is not solid, because x can not equal 2. The part of the number line between -3 and 2 is shaded; that is the section where both inequalities are true.

4. 

$x \ge 2$ OR $x < 1$

Either of the two inequalities can be true, so we are showing the union, or combination, of both. X can be greater than or equal to 2, so the endpoint at 2 is solid. X can also be less than (but not equal to) 1, so that endpoint is not solid. The shaded portion of the number line reflects that either the upper end OR the lower end can make the statements true.

5. $x - 3 \le 4$
$\underline{\quad +3 \quad +3}$
$x \quad \le 7$

Add 3 to each side to isolate x by itself on the left side of the inequality. The result is $x \le 7$. Since the variable can be equal to 7, we can use it to check our results.

$x - 3 \le 4$ — Rewrite the original inequality.
$7 - 3 \le 4$ — Substitute 7 for x.
$4 \le 4$ — After adding, the solution is true. 4 is less than or equal to 4.

6. $3x + 5 > x - 1$

$\underline{-x \qquad -x}$ — Begin by combining like terms; subtract x from both sides.
$2x + 5 > -1$
$\underline{\quad -5 \qquad -5}$ — Undo the operations on the same side as the variable; subtract 5.
$2x \quad > -6$
$\underline{\div 2 \qquad \div 2}$ — Divide both sides by 2 to isolate x by itself.
$x \quad > -3$ — The result is $x > -3$. We can not use -3 to check the result, we must choose

a number larger than -3, so we can use 1.

$3x + 5 > x - 1$ — Rewrite the original inequality.
$3(1) + 5 > 1 - 1$ — Substitute 1 for x. We could check our results with any number > -3.
$3 + 5 > 0$
$8 > 0$ — After following PEMDAS, we get $8 > 0$, which is true.

7. $5x - 2 \le 12 - 2x$

$+2x \quad\quad +2x$	Combine like terms; add 2x to both sides.
$7x - 2 \le 12$	
$+2 \quad +2$	Undo the operations on the same side as the variable; add 2.
$7x \le 14$	
$\div 7 \quad \div 7$	Undo multiplication with division by 7.
$x \le 2$	x can be less than or equal to 2, so we can use 2 to check our results.

$5x - 2 \le 12 - 2x$	Rewrite the original inequality.
$5(2) - 2 \le 12 - 2(2)$	Replace x with 2. We could also use any number < 2.
$10 - 2 \le 12 - 4$	
$8 \le 8$	The result is $8 \le 8$, which is true.

8. $9 - 3x > 15$

$-9 \quad\quad -9$	Subtract 9 from both sides.
$-3x > 6$	
$\div -3 \quad \div -3$	Divide by -3; this means the "greater than" sign will change to "less than."
$x < -2$	The solution is for all number less than -2. We can use -3 to check our results.

$9 - 3x > 15$	Rewrite the original inequality.
$9 - 3(-3) > 15$	Substitute -3 for x.
$9 + 9 > 15$	Remember $-3(-3) = 9$; both signs same, so answer is positive.
$18 > 15$	It is true that $18 > 15$.

9. There are two ways to choose the right solution. The first way is to consider each inequality separately: $-1 \le x$ AND $x < 4$.

To state the first inequality with x on the left, also reverse the sign, so $x \ge -1$. This is represented with a solid endpoint at -1, and the number line larger than -1 is shaded.

The second inequality is represented with an open endpoint at 4, and the number line less than 4 is shaded. C is the only solution that shows the intersection, or the overlap, of where both of these inequalities is true.

The second way to choose the right solution is to consider a point in each range given by the possible answers, and the endpoints shown, and to determine whether <u>both</u> inequalities are true for that value. We will examine the endpoints, -1 and 4, and sample numbers in each range, -3, 1, and 5.

Sample Value	$X \ge -1$	$X < 4$	Are BOTH true?
-3	$-3 \ge -1$ NO	$-3 < 4$ YES	No, do not include this range.
-1	$-1 \ge -1$ YES	$-1 < 4$ YES	YES, include this point.
1	$1 \ge -1$ YES	$1 < 4$ YES	YES, include this range.
4	$4 \ge -1$ YES	$4 < 4$ NO	No, do not include this point.
5	$5 \ge -1$ YES	$5 < 4$ NO	No, do not include this range.

Again, C is the only solution that includes the ranges and the endpoints where BOTH inequalities are true.